Quadratic Equation Questions for NMAT

Instructions

For the following questions answer them individually

Question 1

The number of integers that satisfy the equality $(x^2-5x+7)^{x+1}=1$ **is**

A 3

B 2

C 4

D 5

Answer: A

Explanation:

$(x^2-5x+7)^{x+1}=1$

There can be a solution when $(x^2-5x+7)=1$ or $x^2-5x\ +6=0$

or x=3 and x=2

There can also be a solution when x+1 = 0 or x=-1

Hence three possible solutions exist.

Question 2

The number of distinct real roots of the equation $(x+\frac{1}{x})^2-3(x+\frac{1}{x})+2=0$ **equals**

Answer:1

Explanation:

Let $a=x+\frac{1}{x}$

So, the given equation is $a^2-3a+2=0$

So, a can be either 2 or 1.

If $a=1, x+\frac{1}{x}=1$ and it has no real roots.

If $a=2, x+\frac{1}{x}=2$ and it has exactly one real root which is $x=1$

So, the total number of distinct real roots of the given equation is 1

Question 3

How many disticnt positive integer-valued solutions exist to the equation $(X^2-7x+11)^{(X^2-13x+42)}=1$ **?**

A 8

B 4

C 2

D 6

Answer: D

Explanation:

$(X^2-7x+11)^{(X^2-13x+42)}=1$

if $(X^2-13x+42)$=0 or $(X^2-7x+11)$=1 or $(X^2-7x+11)$=-1 and $(X^2-13x+42)$ is even number

For X=6,7 the value $(X^2-13x+42)$=0

$(X^2-7x+11)$=1 for X=5,2.

$(X^2 - 7x + 11)$=-1 for X=3,4 and for X=3 or 4, $(X^2 - 13x + 42)$ is even number.

∴ {2,3,4,5,6,7} is the solution set of X.

∴ X can take six values.

Question 4

If $x^2 + x + 1 = 0$, then $x^{2018} + x^{2019}$ equals which of the following:

A $x + 1$

B x

C $-x$

D $x - 1$

Answer: C

Explanation:
We know that,

$x^3 - 1 = (x - 1)(x^2 + x + 1)$

Since, $x^2 + x + 1 = 0$

$\therefore x^3 - 1$ = 0

=> x^3 = 1

Now, $x^{2018} + x^{2019}$

= $(x^3)^{672} * x^2$ + $(x^3)^{673}$

= $1^{672} * x^2$ + 1^{673}

= x^2 + 1

= -x

Hence, option C.

Question 5

If $U^2 + (U - 2V - 1)^2$= $-4V(U + V)$, then what is the value of $U + 3V$?

A 0

B $\frac{1}{2}$

C $\frac{-1}{4}$

D $\frac{1}{4}$

Answer: C

Explanation:
Given that $U^2 + (U - 2V - 1)^2$= $-4V(U + V)$

$\Rightarrow U^2 + (U - 2V - 1)(U - 2V - 1)$= $-4V(U + V)$

$\Rightarrow U^2 + (U^2 - 2UV - U - 2UV + 4V^2 + 2V - U + 2V + 1)$ = $-4V(U + V)$

$\Rightarrow U^2 + (U^2 - 4UV - 2U + 4V^2 + 4V + 1)$ = $-4V(U+V)$
$\Rightarrow 2U^2 - 4UV - 2U + 4V^2 + 4V + 1 = -4UV - 4V^2$
$\Rightarrow 2U^2 - 2U + 8V^2 + 4V + 1 = 0$
$\Rightarrow 2[U^2 - U + \frac{1}{4}] + 8[V^2 + \frac{V}{2} + \frac{1}{16}] = 0$
$\Rightarrow 2(U - \frac{1}{2})^2 + 8(V + \frac{1}{4})^2 = 0$

Sum of two square terms is zero i.e. individual square term is equal to zero.

$U - \frac{1}{2}$ = 0 and $V + \frac{1}{4}$ = 0

U = $\frac{1}{2}$ and V = $-\frac{1}{4}$

Therefore, $U + 3V$ = $\frac{1}{2}$+ $\frac{-1*3}{4}$ = $\frac{-1}{4}$. Hence, option C is the correct answer.

Question 6

If $x + 1 = x^2$ and $x > 0$, then $2x^4$ is

A $6 + 4\sqrt{5}$

B $3 + 3\sqrt{5}$

C $5 + 3\sqrt{5}$

D $7 + 3\sqrt{5}$

Answer: D

Explanation:
We know that $x^2 - x - 1 = 0$
Therefore $x^4 = (x+1)^2 = x^2 + 2x + 1 = x + 1 + 2x + 1 = 3x + 2$
Therefore, $2x^4 = 6x + 4$

We know that $x > 0$ therefore, we can calculate the value of x to be $\frac{1+\sqrt{5}}{2}$
Hence, $2x^4 = 6x + 4 = 3 + 3\sqrt{5} + 4 = 3\sqrt{5} + 7$

Question 7

If x^2+3x-10is a factor of $3x^4 + 2x^3 - ax^2 + bx - a + b - 4$ then the closest approximate values of a and b are

A 25, 43

B 52, 43

C 52, 67

D None of the above

Answer: C

Explanation:
If x^2+3x-10is a factor of $3x^4 + 2x^3 - ax^2 + bx - a + b - 4$
Then x = -5 and x = 2 will give $3x^4 + 2x^3 - ax^2 + bx - a + b - 4$ = 0
Substituting x = -5 we get,

$3(-5)^4+2(-5)^3-a(-5)^2+b(-5)-a+b-4=0$
Solving we get,
$26a+4b=1621$.......(i)
Substituting x = 2 we get,
$3(2)^4+2(2)^3-a(2)^2+b(2)-a+b-4=0$
=> $5a-3b=60$........(ii)
Solving i and ii we get
a and b $\approx 52, 67$
Hence, option C is the correct answer.

Question 8

If $xy+yz+zx=0$, then $(x+y+z)^2$ equals

A $(x+y)^2+xz$

B $(x+z)^2+xy$

C $x^2+y^2+z^2$

D $2(xy+yz+xz)$

Answer: C

Explanation:
$(x+y+z)^2=x^2+y^2+z^2+2(xy+yz+xz)$
as $xy+yz+xz=0$
so equation will be resolved to $x^2+y^2+z^2$

Question 9

If the equation $x^3-ax^2+bx-a=0$ has three real roots, then it must be the case that,

A b=1

B b $\neq$ 1

C a=1

D a $\neq$ 1

Answer: B

Explanation:
It can be clearly seen that if b=1 then $x^2(x-a)+(x-a)=0$ an the equation gives only 1 real value of x

Question 10

If the roots of the equation $x^3-ax^2+bx-c=0$ are three consecutive integers, then what is the smallest possible value of b? [CAT 2008]

A $\frac{-1}{\sqrt{3}}$

B -1

C 0

D 1

E $\frac{1}{\sqrt{3}}$

Answer: B

Explanation:
b = sum of the roots taken 2 at a time.
Let the roots be n-1, n and n+1.
Therefore, $b = (n-1)n + n(n+1) + (n+1)(n-1) = n^2 - n + n^2 + n + n^2 - 1$
$b = 3n^2 - 1$. The smallest value is -1.