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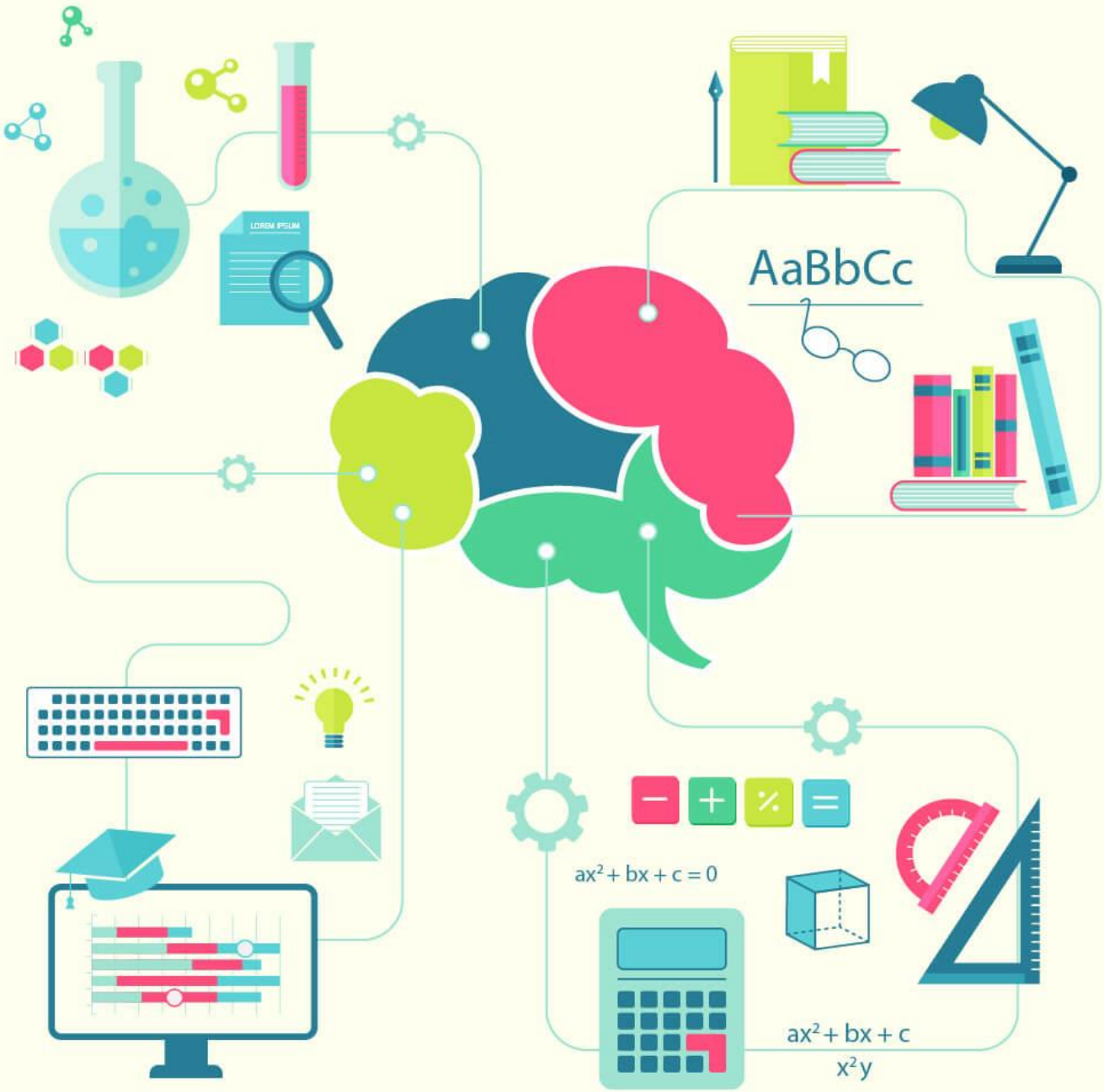
Euler's Theorem for CAT



Euler's totient

Euler's theorem is one of the most important remainder theorems. It is imperative to know about Euler's totient before we can use the theorem.

Euler's totient is defined as the number of numbers less than 'n' that are co-prime to it.
It is usually denoted as $\phi(n)$.

The formula to find Euler's totient is $\phi(n) = n*(1 - \frac{1}{a}) * (1 - \frac{1}{b})*\ldots$ where a, b are the prime factors of the numbers.

Eg) Find the number of numbers that are less than 30 and are co-prime to it.

30 can be written as 2*3*5.
$\phi(30) = 30 * 1/2 * 2/3 * 4/5$
$= 8$

Therefore, 8 numbers less than 30 are co-prime to it.

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Euler's Theorem

Euler's theorem states that $a^{\phi(n)}$ (mod n) = 1 (mod n) if 'a' and 'n' are co-prime to each other.
So, if the given number 'a' and the divisor 'n' are co-prime to each other, we can use Euler's theorem.

Example 1:
What is the remainder when 2^{256} is divided by 15?
2 and 15 are co-prime to each other. Hence, Euler's theorem can be applied.
15 can be written as 5*3.

Euler's totient of 15 = $15*(1 - \frac{1}{3})*(1 - \frac{1}{5}) = 15*\frac{2}{3}*\frac{4}{5} = 8$

Therefore, we have to try to express 256 as 8k + something. 256 can be expressed as 8*32

We know that, $a^{\phi(n)}$ (mod n) = 1 (mod n)
2^{8*32} (mod 15) = 1 (mod 15).

Therefore, 1 is the right answer.

Example 2:
What are the last 2 digits of 7^{2008} ?

Finding the last 2 digits is similar to finding the remainder when the number is divided by 100.
100 and 7 are co-prime to each other. Hence, we can use Euler's theorem.

100 can be written as $2^2 * 5^2$.
Euler's totient of 100, $\phi(100) = 100*(1 - \frac{1}{2}) * (1 - \frac{1}{5})$.

$$= 100* (\frac{1}{2}) * (\frac{4}{5})$$

$$\phi(100) = 40.$$

7^{2008} can be written as $7^{2000}*7^8$

7^{2000} can be written as $7^{40*(25)}$. Hence, 7^{2000} will yield a remainder of 1 when divided by 100.
The problem is reduced to what will be the remainder when 7^8 is divided by 100.
We know that $7^4 = 2401$.
$7^8 = 7^4*7^4 = 2401*2401$.

As we can clearly see, the last 2 digits will be 01.

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