

Number Systems Tips by CRACKU.IN

- Number Systems is the most important topic in the quantitative section.
- It is a very vast topic and a significant number of questions appear in CAT every year from this section.
- Learning simple tricks like divisibility rules, HCF and LCM, prime number and remainder theorems can help improve the score drastically.
- This document presents best short cuts which makes this topic easy and helps you perform better.

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Cracku Tip 1 - Number systems

HCF and LCM

- $\text{HCF} * \text{LCM}$ of two numbers = Product of two numbers
- The greatest number dividing a , b and c leaving remainders of x_1 , x_2 and x_3 is the HCF of $(a-x_1)$, $(b-x_2)$ and $(c-x_3)$.
- The greatest number dividing a , b and c ($a < b < c$) leaving the same remainder each time is the HCF of $(c-b)$, $(c-a)$, $(b-a)$.
- If a number, N , is divisible by X and Y and $\text{HCF}(X, Y) = 1$. Then, N is divisible by $X * Y$

Cracku Tip 2 - Number systems

Prime and Composite Numbers

- Prime numbers are numbers with only two factors, 1 and the number itself.
- Composite numbers are numbers with more than 2 factors. Examples are 4, 6, 8, 9.
- 0 and 1 are neither composite nor prime.
- There are 25 prime numbers less than 100.

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Cracku Tip 3 - Number systems

Properties of Prime numbers

- To check if n is a prime number, list all prime factors less than or equal to \sqrt{n} . If none of the prime factors can divide n then n is a prime number.
- For any integer a and prime number p , $a^p - a$ is always divisible by p
- All prime numbers greater than 2 and 3 can be written in the form of $6k+1$ or $6k-1$
- If a and b are co-prime then $a^{(b-1)} \bmod b = 1$.

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Cracku Tip 4 - Number systems

Theorems on Prime numbers

Fermat's Theorem:

Remainder of $a^{(p-1)}$ when divided by p is 1, where p is a prime

Wilson's Theorem:

Remainder when $(p-1)!$ is divided by p is $(p-1)$ where p is a prime

Cracku Tip 5 - Number systems

Theorems on Prime numbers

Remainder Theorem

- If a, b, c are the prime factors of N such that $N = a^p * b^q * c^r$
Then the number of numbers less than N and co-prime to N is
$$\phi(N) = N (1 - 1/a) (1 - 1/b) (1 - 1/c).$$

This function is known as the Euler's totient function.

Euler's theorem

- If M and N are co-prime to each other then remainder when $M^{\phi(N)}$ is divided by N is 1.

Cracku Tip 6 - Number systems

- Highest power of n in $m!$ is $[m/n] + [m/n^2] + [m/n^3] + \dots$

Ex: Highest power of 7 in $100! = [100/7] + [100/49] = 16$

- To find the number of zeroes in $n!$ find the highest power of 5 in $n!$
- If all possible permutations of n distinct digits are added together the sum = $(n-1)! * (\text{sum of } n \text{ digits}) * (11111\dots n \text{ times})$

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Cracku Tip 7 - Number systems

- If the number can be represented as $N = a^p * b^q * c^r \dots$ then number of factors the is $(p+1) * (q+1) * (r+1)$
- Sum of the factors = $\frac{a^{p+1} - 1}{a - 1} * \frac{b^{q+1} - 1}{b - 1} * \frac{c^{r+1} - 1}{c - 1}$
- If the number of factors are odd then N is a perfect square.
- If there are n factors, then the number of pairs of factors would be $n/2$. If N is a perfect square then number of pairs (including the square root) is $(n+1)/2$

Cracku Tip 8 - Number systems

If the number can be expressed as $N = 2^p * a^q * b^r \dots$ where the power of 2 is p and a, b are prime numbers

- Then the number of even factors of $N = p (1+q) (1+r) \dots$
- The number of odd factors of $N = (1+q) (1+r) \dots$

Cracku Tip 9 - Number systems

Number of positive integral solutions of the equation $x^2 - y^2 = k$ is given by

- $\frac{\text{Total number of factors of } k}{2}$ (If k is odd but not a perfect square)
- $\frac{(\text{Total number of factors of } k) - 1}{2}$ (If k is odd and a perfect square)
- $\frac{\text{Total number of factors of } \frac{k}{4}}{2}$ (If k is even and not a perfect square)
- $\frac{(\text{Total number of factors of } \frac{k}{4}) - 1}{2}$ (If it is even and a perfect square)

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Cracku Tip 10 - Number systems

- Number of digits in $a^b = [b \log_m(a)] + 1$; where m is the base of the number and $[.]$ denotes greatest integer function
- Even number which is not a multiple of 4, can never be expressed as a difference of 2 perfect squares.

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Cracku Tip 11 - Number systems

- Sum of first n odd numbers is n^2
- Sum of first n even numbers is $n(n+1)$
- The product of the factors of N is given by $N^{a/2}$, where a is the number of factors



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Cracku Tip 12 - Number systems

- The last two digits of a^2 , $(50 - a)^2$, $(50+a)^2$, $(100 - a)^2$ are same.
- If the number is written as 2^{10n}

When n is odd, the last 2 digits are 24.

When n is even, the last 2 digits are 76.

Cracku Tip 13 - Number systems

Divisibility

- Divisibility by 2: Last digit divisible by 2
- Divisibility by 4: Last two digits divisible by 4
- Divisibility by 8: Last three digits divisible by 8
- Divisibility by 16: Last four digit divisible by 16

Cracku Tip 14 - Number systems

Divisibility

- Divisibility by 3: Sum of digits divisible by 3
- Divisibility by 9: Sum of digits divisible by 9
- Divisibility by 27: Sum of blocks of 3 (taken right to left) divisible by 27
- Divisibility by 7: Remove the last digit, double it and subtract it from the truncated original number. Check if number is divisible by 7
- Divisibility by 11: (sum of odd digits) - (sum of even digits) should be 0 or divisible by 11

Cracku Tip 15 - Number systems

Divisibility properties

- For composite divisors, check if the number is divisible by the factors individually. Hence to check if a number is divisible by 6 it must be divisible by 2 and 3.
- The equation $a^n - b^n$ is always divisible by $a - b$. If n is even it is divisible by $a + b$. If n is odd it is not divisible by $a + b$.
- The equation $a^n + b^n$, is divisible by $a + b$ if n is odd. If n is even it is not divisible by $a + b$.

Cracku Tip 16 - Number systems

- Converting from decimal to base b . Let $R_1, R_2 \dots$ be the remainders left after repeatedly dividing the number with b . Hence, the number in base b is given by $\dots R_2R_1$.
- Converting from base b to decimal - multiply each digit of the number with a power of b starting with the rightmost digit and b^0 .
- A decimal number is divisible by $b-1$ only if the sum of the digits of the number when written in base b are divisible by $b-1$.

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Cracku Tip 17 - Number systems

Cyclicity

▶ To find the last digit of a^n find the cyclicity of a . For Ex. if $a=2$, we see that

▶ $2^1=2$

▶ $2^2=4$

▶ $2^3=8$

▶ $2^4=16$

▶ $2^5=32$

Hence, the last digit of 2 repeats after every 4th power. Hence cyclicity of 2 = 4. Hence if we have to find the last digit of a^n ,

The steps are:

1. Find the cyclicity of a , say it is x
2. Find the remainder when n is divided by x , say remainder r
3. Find a^r if $r>0$ and a^x when $r=0$

Cracku Tip 18 - Number systems

- $(a + b)(a - b) = (a^2 - b^2)$
- $(a + b)^2 = (a^2 + b^2 + 2ab)$
- $(a - b)^2 = (a^2 + b^2 - 2ab)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$



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Cracku Tip 19 - Number systems

- $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
- $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
- $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
- When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

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