CAT Questions on Prime Numbers
Instructions
For the following questions answer them individually

Question 1
If \( n = 1 + x \), where \( x \) is the product of four consecutive positive integers, then which of the following is/are true?

A. \( n \) is odd
B. \( n \) is prime
C. \( n \) is a perfect square

A. A and C only
B. A and B only
C. A only
D. None of these

Answer: A

Explanation:
Let the four consecutive positive integers be \( a, a+1, a+2 \) and \( a+3 \).
Therefore, \( n = 1 + a(a+1)(a+2)(a+3) \)
Or, \( n = 1 + (a^2 + 3a) + (a^2 + 3a + 2) \)
Or, \( n = (a^2 + 3a)^2 + 2 \cdot (a^2 + 3a) + 1 = (a^2 + 3a + 1)^2 \)

Hence, \( n \) is a perfect square and therefore not a prime.

The product of four consecutive positive integers is always even. Hence, \( n \) is always odd.
Therefore, from the given statements, only A and C are true.

Question 2
Let \( X \) be a four-digit positive integer such that the unit digit of \( X \) is prime and the product of all digits of \( X \) is also prime. How many such integers are possible?

A. 4
B. 8
C. 12
D. 24
E. None of the above

Answer: A

Explanation:
Given that unit digit of \( X \) and product of all 4 digits of \( X \) are prime.

The product of two numbers to be prime is possible only when one of the numbers is prime and the other is ‘1’.

The possibilities for the prime unit digits are - 2, 3, 5, 7 (Note that 1 is not a prime number)

Hence the possibility of remaining 3 digits, considering the product of all 4 digits to be prime is ‘111’ only.

Hence all the possible numbers are 1112, 1113, 1115, 1117

\( \therefore \) Total 4 integers are possible.

Question 3
Let \( S \) be the set of prime numbers greater than or equal to 2 and less than 100. Multiply all the elements of \( S \). With how many consecutive zeroes will the product end?

\[ n = 1 + a(a+1)(a+2)(a+3) \]
\[ n = 1 + (a^2 + 3a) + (a^2 + 3a + 2) \]
\[ n = (a^2 + 3a)^2 + 2 \cdot (a^2 + 3a) + 1 = (a^2 + 3a + 1)^2 \]
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**Question 4**
Consider four natural numbers: x, y, x + y, and x - y. Two statements are provided below:
I. All four numbers are prime numbers.
II. The arithmetic mean of the numbers is greater than 4.
Which of the following statements would be sufficient to determine the sum of the four numbers?

A. Statement I.
B. Statement II.
C. Statement I and Statement II.
D. Neither Statement I nor Statement II.
E. Either Statement I or Statement II.

**Answer:** A

**Explanation:**
Natural numbers = x, y, (x + y), (x - y)
Statement I: As all the numbers are prime, therefore, either x or y has to be 2 because otherwise (x+y) cannot be prime.
Case 1: If x = 2, then (x-y) cannot be prime
Case 2: If y = 2, numbers = (x - 2), x, (x + 2)
These numbers are prime, hence all possibility = 3,5,7
∴ Sum = 2+3+5+7 = 17
Using statement II, we cannot find the required sum, as no specific value of mean is given.
Thus, statement I alone is sufficient.

**Question 5**
There are four prime numbers written in ascending order of magnitude. The product of the first three is 7429 and last three is 12673. Find the first number.

A. 19
B. 17
C. 13
Question 6
The number of employees in Obelix Menhir Co. is a prime number and is less than 300. The ratio of the number of employees who are graduates and above, to that of employees who are not, can possibly be:

A 101:88
B 87:100
C 110:111
D 85:98
E 97:84

Answer: E

Explanation:
The addition of numerator and denominator should give a prime no. Only option E gives that.
3 is a factor of 189 and 183 => A and D eliminated
17 is a factor of 187 and 221 => B and C eliminated
181 is prime.

Question 7
A positive integer is said to be a prime number if it is not divisible by any positive integer other than itself and 1. Let \( p \) be a prime number greater than 5. Then \( (p^2 - 1) \) is

A never divisible by 6.
B always divisible by 6, and may or may not be divisible by 12.
C always divisible by 12, and may or may not be divisible by 24.
D always divisible by 24.

Answer: D

Explanation:
Let the Prime number be 6n+1.

So \((p^2 - 1) = 6n(6n+2) = 12n(3n+1)\)

For any value of \(n\), \(n(3n+1)\) will have a factor of 2

Hence given equation will be always be divisible by 24

Question 8

Z is the product of first 31 natural numbers. If \(X = Z + 1\), then the numbers of primes among \(X + 1, X + 2, ..., X + 29, X + 30\) is

A 30
B 2
C Cannot be determined
D None of the above

Answer: D

Explanation:
It is given that \(Z = 31!\)
\(X = 31! + 1\)
\(X+1 = 31!+2\) this is divisible by 2
\(X+2 = 31!+3\) this is divisible by 3
\(X+3 = 31!+4\) this is divisible by 4
...
\(X+29 = 31!+30\) this is divisible by 30
\(X+30 = 31!+31\) this is divisible by 31

Hence, none of \(X + 1, X + 2, ..., X + 29, X + 30\) is a prime number.
Hence, option D is the correct answer.

Question 9

Please read the following sentences carefully:
I — 103 and 7 are the only prime factors of 1000027
II — \(\sqrt{6!} > \sqrt{7!}\)
III — If I travel one half of my journey at an average speed of \(x\) km/h, it will be impossible for me to attain an average speed of \(2x\) km/h for the entire journey.

A All the statements are correct
B Only Statement II is correct
C Only Statement III is correct
D Both statements I and II are correct
E Both statements I and III are correct

Answer: C

Explanation:
Let us evaluate the statements one by one:
I: 103 and 7 are the only prime factors of 1000027
On successively dividing 1000027 by 103 and 7, we get 1387 as the answer.
1387 is divisible by 19.
Therefore, statement I is false.
II: \(\sqrt{6!} > \sqrt{7!}\)
Raising the power to 42 on both sides, we get,

\[ (6!)^7 > (7!)^6 \]

\[ 6! \times (6!)^6 > 7^6 \times (6!)^6 \]

\[ 7^6 \text{ is greater than } 6!. \]

Therefore, statement II is false.

III: It has been given that the person travels one-half of the journey at x kmph. Let us assume the distance to be '2d'.
Let us assume the average speed to be 2d and check for the feasibility.
Let the speed at which the person travels the other half of the journey be y.
\[ \frac{d}{x} + \frac{d}{y} = \frac{2d}{2x} \]
\[ \frac{d}{x} + \frac{d}{y} = \frac{d}{x} \]
\[ \Rightarrow \frac{d}{y} = 0 \text{ or } y \text{ tends to infinity. } \]
Therefore, such a scenario is not possible and hence, statement III is true.
Only statement III is true. Therefore, option C is the right answer.
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