Top-100 Most Expected CAT Questions and Answers

Instructions

For the following questions answer them individually

Question 1

A regular octagon PQRSTUVW is inscribed inside a circle of radius 'R'. A square ABCD is formed by joining mid-points of sides PQ, RS, TU, and VW. Find the ratio of the area of the square to that of the circle.

A $\frac{(\sqrt{2}+1)}{\sqrt{2}} : \pi$

B $\frac{2}{1+\sqrt{2}} : \pi$

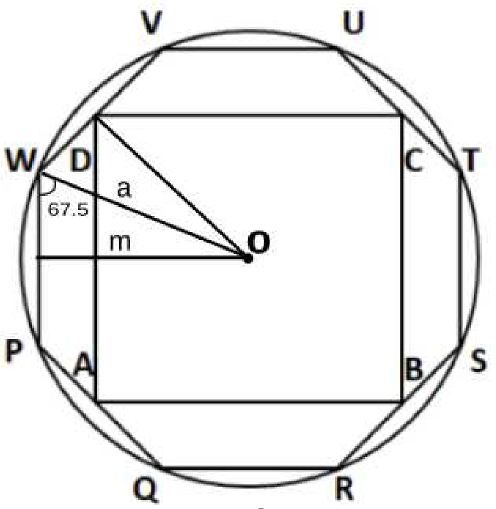
C $\sqrt{2} : \pi$

D $1 : \pi$

E None of these

Answer: A

Explanation:



For an octagon WOP = 45^0

Thus angle PWO= (180-45)/2= 67.5

The radius of the circle = a

Area of the circle = πa^2

OD is the perpendicular from the side of the octagon, which is m.

OD is also 1/2 of the diagonal.

Diagonal to the square =2m

Area of the square= $2m^2$

The ratio of the area of the square to that of the circle.= $2m^2 : \pi a^2$

= $\frac{2}{\pi} \times \left(\frac{a}{m}\right)^2$

$\left(\frac{a}{m}\right)^2$ =$(\sin 67.5)^2$= $(\cos 22.5)^2$

cos 45 = $2(\cos 22.5)^2$-1

$1/\sqrt{2}$ =$2(\cos 22.5)^2$-1

$(\cos 22.5)^2$ = $\frac{(\sqrt{2}+1)}{2\sqrt{2}}$

Answer= $\frac{2}{\pi} \times \left(\frac{a}{m}\right)^2 = \frac{2}{\pi} \frac{(\sqrt{2}+1)}{2\sqrt{2}}$

= $\frac{(\sqrt{2}+1)}{\sqrt{2}} : \pi$

Question 2

A walking track AB is the diameter of a circular park of radius 10 m. A pole of height 6 m is standing on the circumference of the circular park and it subtends equal angles at A and B. A point R lies on the line AB and the pole subtends 30° at R. What is the distance of the point R from the center?

A $2\sqrt{2}$

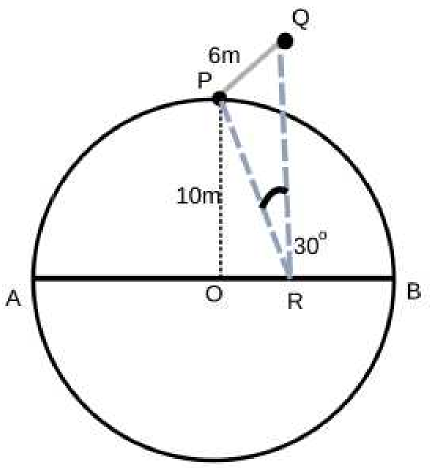
B $\sqrt{12}$

C $\sqrt{15}$

D $3\sqrt{2}$

Answer: A

Explanation:
As given that pole subtends equal angle at the ends of the diameter AB. Then it will be at perpendicular from the center as given in the figure below.



Here suppose the pole PQ subtends 30 $^\circ$ on the point R.

Then PQ/PR= tan 30°

PR=$6\sqrt{3}$

Using Pythagoras theorem:

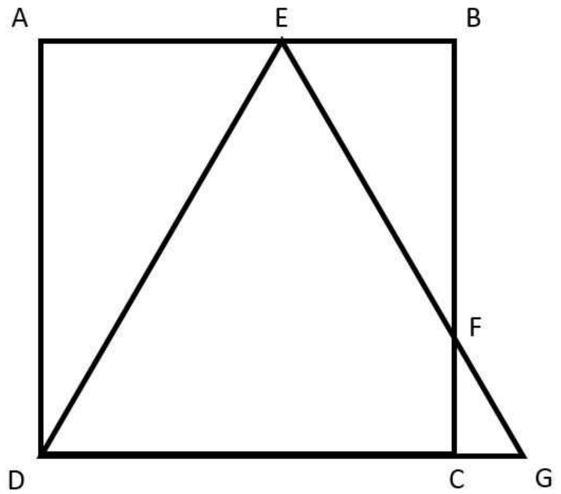
$PR^2 = PO^2 + OR^2$

$36 * 3 = 100 + X^2$

OR = $2\sqrt{2}$

Question 3

If DEG is an equilateral triangle of side 2 cm and ABCD is a square. What is the area of the triangle BEF?



A $3\sqrt{3}-2$

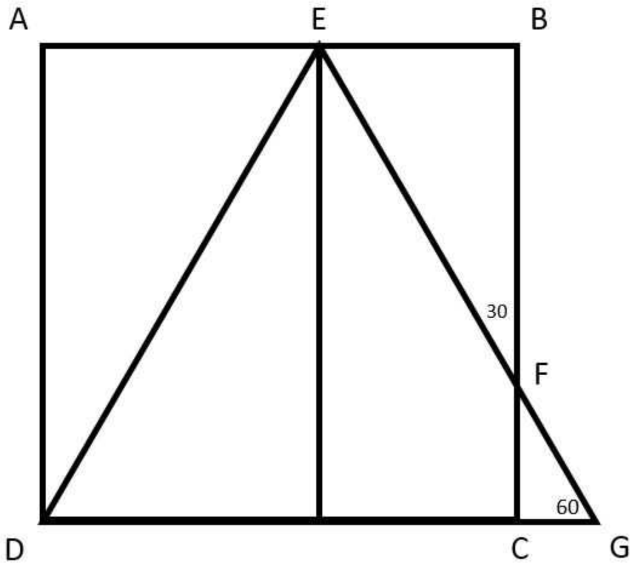
B $3-\sqrt{3}$

C $2\sqrt{3}-3$

D $2+\sqrt{3}$

Answer: C

Explanation:



Side of the square would be equal to the altitude of the equilateral triangle.

Therefore, the side of the square => $\frac{\sqrt{3}}{2} \times 2$ = $\sqrt{3}$ cm

Since, altitude bisects the side => AE = 1 cm

=> EB = $(\sqrt{3}-1)$ cm

=> Area of triangle EBF = $\frac{1}{2}$ x EB x BF

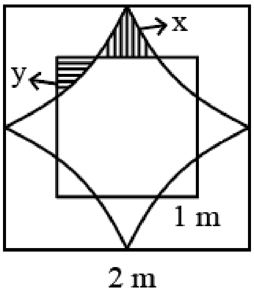
=> Area of triangle EBF = $\frac{1}{2}$ x EB x $\frac{EB}{\tan 30^{\circ}}$

=> Area of triangle EBF = $2\sqrt{3}-3$

Hence, option C

Question 4

A square of length 1 m is inside a square of length 2 m and four quarter circles are joined as shown in the figure. The value of y —x is given by,



A $\frac{8-\pi}{10}$

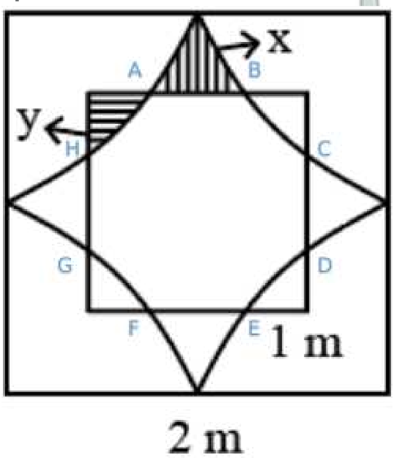
B $\frac{4-\pi}{5}$

C $\frac{2\pi-1}{8}$

D $\frac{\pi-3}{4}$

Answer: D

Explanation:



From the above figure area of the region bounded by ABCDEFGH = Are of the square with side 2 - (4*quadrants with side 1cm + 4x)

=4-π -4x

Which is same as the area of the square with side 1 cm -4y

4-π -4x = 1-4y

4x-4y=3-π

y-x= $\frac{\pi-3}{4}$

D is the correct answer.

Question 5

Points E, F, G, H lie on the sides AB, BC, CD, and DA, respectively, of a square ABCD. If EFGH is also a square whose area is 62.5% of that of ABCD and CG is longer than EB, then the ratio of length of EB to that of CG is

A 3 : 8

B 2 : 5

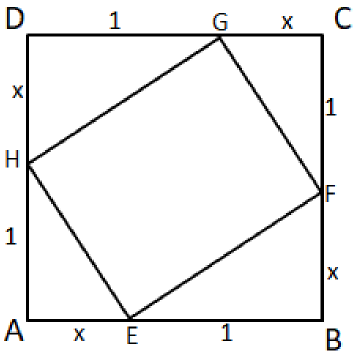
C 4 : 9

D 1 : 3

Answer: D

Explanation:
It is given that EFGH is also a square whose area is 62.5% of that of ABCD. Let us assume that E divides AB in x : 1. Because of symmetry we can se that points F, G and H divide BC, CD and DA in x : 1.

Let us assume that 'x+1' is the length of side of square ABCD.



Area of square ABCD = $(x+1)^2$ sq. units.

Therefore, area of square EFGH = $\frac{62.5}{100} * (x+1)^2$ = $\frac{5(x+1)^2}{8}$ ... (1)

In right angle triangle EBF,

$EF^2 = EB^2 + BF^2$

$\Rightarrow EF = \sqrt{1^2 + x^2}$

Therefore, the area of square EFGH = EF^2 = $x^2 + 1$ ... (2)

By equating (1) and (2),

$x^2 + 1 = \frac{5(x+1)^2}{8}$

$\Rightarrow 8x^2 + 8 = 5x^2 + 10x + 5$

$\Rightarrow 3x^2 - 10x + 3 = 0$

$\Rightarrow (x-3)(3x-1) = 0$

$\Rightarrow x = 3$ or $1/3$

The ratio of length of EB to that of CG = 1 : x

EB : CG = 1 : 3 or 3 : 1. Hence, option D is the correct answer.

Question 6

In a trapezium ABCD, AC and BD are two diagonals which intersect at point O. Points E and F are the midpoints of AC and BD respectively. If AB is parallel to CD and AB:CD = 5:3, then find out the ratio of the area of triangle OEF to that of trapezium ABCD.

A $\frac{1}{64}$

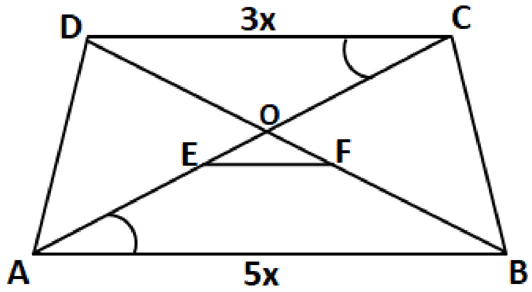
B $\frac{1}{36}$

C $\frac{1}{9}$

D None of the above

Answer: A

Explanation:
Let us the diagram according to the information given in the question.



AB is parallel to DC. Hence, ∠ABD = ∠CDB and ∠CAB = ∠ACD

Therefore, we can say that triangle AOB is similar to triangle COD.

$\frac{AO}{CO} = \frac{BO}{DO} = \frac{AB}{CD}$

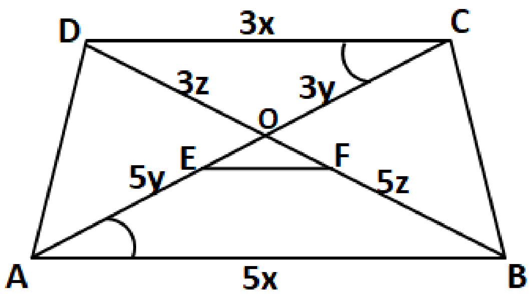
$\Rightarrow \frac{AO}{CO} = \frac{BO}{DO} = \frac{5}{3}$

Let '8y' and '8z' be the lengths of AC and BD respectively.

Then, AO = 5y and OC = 3y. Also, E is the mid point of AC hence OE = y.

Similarly, BO = 5z and OD = 3z. Also, F is the mid point of BD hence OF = z.

Hence, the area of △OEF = $\frac{1}{2} * OE * OF * \sin(EOF)$ = $\frac{yz}{2} * \sin(EOF)$ ... (1)



Area of trapezium ABCD = Area of △AOB + Area of △BOC + area of △COD + area of △DOA

Area of trapezium ABCD = $\frac{1}{2} * AO * BO * \sin(AOB)$ + $\frac{1}{2} * BO * CO * \sin(BOC)$ + $\frac{1}{2} * CO * DO * \sin(COD)$ + $\frac{1}{2} * DO * AO * \sin(DOA)$

We can see that ∠DOC = ∠AOB and ∠COB = ∠AOD

Also, ∠AOB + ∠BOC = π

$\Rightarrow$ ∠BOC = π - ∠AOB

$\Rightarrow \sin(BOC)$ = $\sin(\pi - AOB)$

$\Rightarrow \sin(BOC) = \sin(AOB)$

Therefore, we can say that,

$\sin(BOC)$ = $\sin(AOB)$ = $\sin(COD) = \sin(DOA)$.

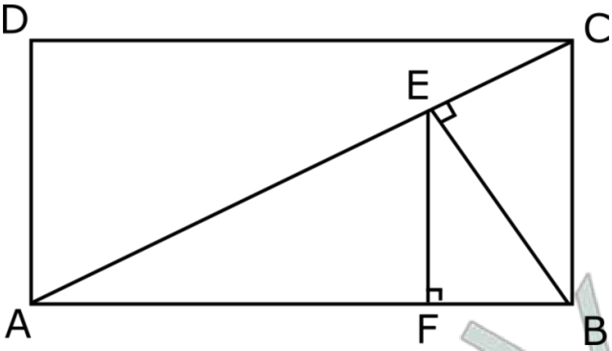
Area of trapezium ABCD = $\frac{25xy + 15xy + 9xy + 15xy}{2} * \sin(AOB)$ = $32xy * \sin(AOB)$

Therefore, the ratio of the area of triangle OEF to that of trapezium ABCD = $\frac{\frac{yz}{2} * \sin(EOF)}{32xy * \sin(AOB)}$ = $\frac{1}{64}$

Hence, option A is the correct answer.

Question 7

ABCD is a rectangle as shown in the figure. AB = 8 cm and BC = 6 cm. BE is the perpendicular drawn from B to the diagonal AC. EF is the perpendicular drawn from E to AB. What is the length of BF?



A 3.24 cm

B 1.96 cm

C 2.56 cm

D 2.88 cm

Answer: D

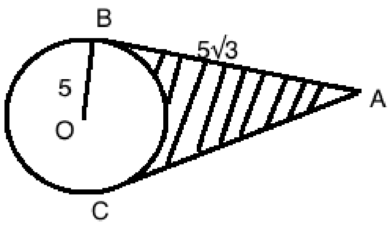
Explanation:
By Pythagoras theorem $AC = \sqrt{AB^2 + BC^2} = \sqrt{8^2 + 6^2} = 10$ cm

Triangle BEC is similar to triangle ABC. So we get,
$\frac{BE}{AB} = \frac{BC}{AC}$
=> $\frac{BE}{8} = \frac{6}{10}$
=> $BE = 4.8$ cm

Triangle BFE is similar to triangle BEA. So we get,
$\frac{BF}{BE} = \frac{BE}{AB}$
=> $BF = \frac{BE^2}{AB}$
=> $BF = \frac{4.8^2}{8}$
=> $BF = 2.88$ cm

Question 8

AB and AC are tangents to the circle with centre O. If the radius of the circle is 5 and length of the tangent is $5\sqrt{3}$, what is the area of the shaded region?



A $25(\sqrt{3} - \frac{\pi}{3})$

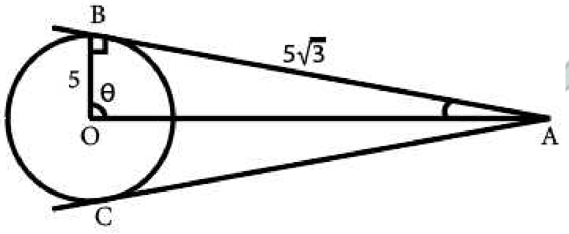
B $25(\frac{\sqrt{3}}{2} - \frac{\pi}{4})$

C $25(\sqrt{3} - \frac{\pi}{4})$

D $25(\sqrt{3} - \frac{\pi}{6})$

Answer: A

Explanation:



In the triangle BOA, tan $\theta = \frac{5\sqrt{3}}{5} = \sqrt{3}$
Therefore, $\theta = 60$ degrees.

The shaded area = Area of two triangles - area of section of circle.
As the radius is perpendicular to a tangent, triangle OAB is a right angled triangle, with right angle at B, with area=$1/2 * 5 * 5\sqrt{3}$.
Hence area of both triangles= $2 * 1/2 * 5 * 5\sqrt{3} = 25\sqrt{3}$.

The angle BOA=$tan^{-1}\frac{5\sqrt{3}}{5} = 60^{\circ}$.

Hence, angle BOC = 2*BOA = 120°

Hence the area of the section = area of circle/3 = $\frac{\pi(5^2)}{3}$.
Hence area of shaded portion= $25(\sqrt{3} - \frac{\pi}{3})$

Question 9

In a rectangle PQRS, X and Y are two points on the sides PQ and QR respectively. If the areas of $\triangle$XYQ, $\triangle$SPX and $\triangle$SRY are in a ratio of 4 : 3 : 10, then find out the ratio of the area of triangle SXY to the area of the rectangle PQRS.

A $\frac{11}{30}$

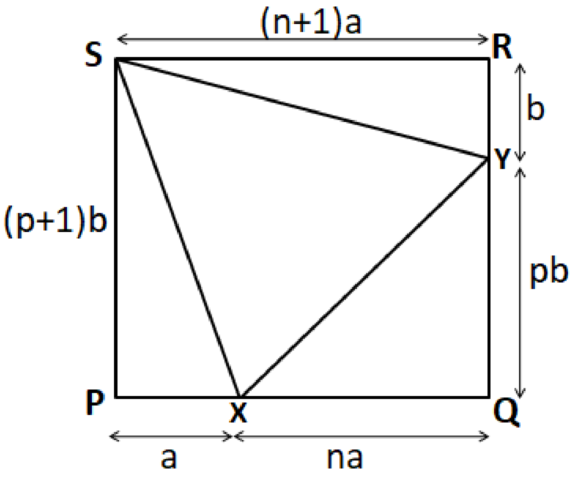
B $\frac{8}{15}$

C $\frac{7}{15}$

D $\frac{13}{30}$

Answer: D

Explanation:
Let us assume that X divides PQ in 1:n and Y divides QR in p:1.We can draw the diagram as:



It is given that the areas of △XYQ, △SPX and △SRY are in a ratio of 4 : 3 : 10.

Considering △XYQ and △SPX

$$\Rightarrow \frac{\text{Area of triangle } XYQ}{\text{Area of triangle } SPX} = \frac{4}{3}$$

$$\Rightarrow \frac{1/2 * pb * na}{1/2 * a * (p+1)b} = \frac{4}{3}$$

$$\Rightarrow \frac{np}{p+1} = \frac{4}{3}$$ ... (1)

Considering △XYQ and △SRY

$$\Rightarrow \frac{\text{Area of triangle } XYQ}{\text{Area of triangle } SRY} = \frac{4}{10}$$

$$\Rightarrow \frac{1/2 * pb * na}{1/2 * a * (n+1)b} = \frac{2}{5}$$

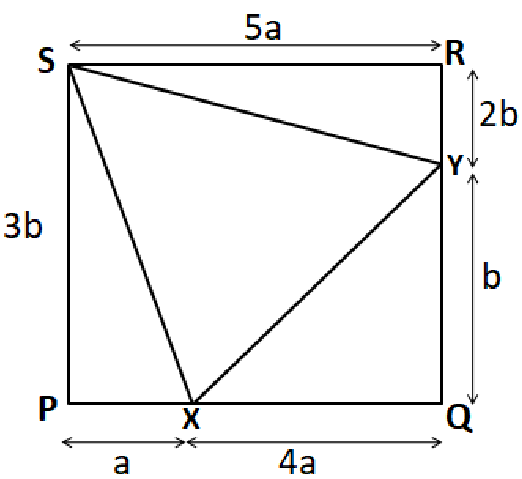
$$\Rightarrow \frac{np}{n+1} = \frac{2}{5}$$ ... (2)

From equation (1) and (2)

$$\Rightarrow \frac{2n + 2n^2}{2 + 7n} = \frac{4}{3}$$

⇒ n = 4 or -1/3. $n \neq -1/3$ therefore n = 4.

Also p = 1/2. Hence we can say that X divides PQ in 1 : 4 whereas Y divides QR in 1 : 2.



Therefore, the area of triangle SXY = Area of rectangle PQRS - Area of △XYQ - Area of △SPX - Area of △SRY

$\Rightarrow 5a * 3b - 24a * b - 2a * 3b - 25a * 2b$

The area of triangle SXY = $(13/2)ab$

Hence, the ratio of the area of triangle SXY to the area of the rectangle PQRS. = $\frac{(13/2)ab}{15ab} = \frac{13}{30}$.

Therefore, option D is the correct answer.

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Question 10

If three sides of a rectangular park have a total length 400 ft, then the area of the park is maximum when the length (in ft) of its longer side is

Answer:200

Explanation:
Let the length and breadth of the park be l,b, l > b
Case 1: 2l + b = 400
Area = lb. Area is maximum when 2l * b is maximum, which is maximum when 2l = b (using AM $\geq$ GM inequality) => l = 100, b = 200.
Which can't happen since l > b

Case 2: l + 2b = 400
Area = lb. Area is maximum when l *2 b is
maximum, which is maximum when l = 2b (using AM $\geq$ GM
inequality) => l = 200, b = 100.

Hence length of the longer side is 200 ft

Question 11

Ramesh and Suresh decide to run on a circular track. They run in the same direction and the point at which they meet for 7th time is same as the point at which they meet for the 25th time. If the ratio of the speeds of Ramesh and Suresh is k:1, where k is a natural number, then for how many values of k is the given situation possible?

Answer:6

Explanation:
Ramesh and Suresh are running in the same direction and the ratio of their speed is k:1, so they will meet at k-1 distinct points. If L is the length of the track, then these points would be $\frac{L}{K-1}$, $\frac{2L}{K-1}$, $\frac{3L}{K-1}$ .....L. They meet at each of these points once before they meet at the same point twice. Since 7th meeting point is same as 25th, the number of distinct meeting points is 25-7 = 18. So for all the values of k for which k-1 is a factor of 18, their 7th and 25th meeting point would be same. Factors of 18 are 1,2,3,6,9,18, so possible values of k are 2,3,4,7,10,19. So there are 6 values of k which are possible.

Question 12

Kookaburra Sport ltd. manufactures cricket bats and balls. All the bats are identical hence it takes the same amount of time in the manufacturing each bat. The same is true for balls as well. All the workers who are employed in Kookaburra Sport ltd are equally efficient. In one day, 20 workers can produce 60 bats and 40 balls. In two days, 45 workers can produce 180 bats and 225 balls. In six days, 25 workers can manufacture 'm' bats and 'n' balls. If the value of |m - n| is the minimum, then find out the value of (m+n).

Answer:700

Explanation:
It is given that all workers are equally efficient hence they must take the same amount of time to manufacture one bat and same is true for a ball as well.
Let 'x' and 'y' be the number of days that 1 worker takes in manufacturing one bat and one ball.
Also, 45 workers in two days will accomplish the same amounts of work as 90 workers in one day.
Hence, we can say that,
$\Rightarrow 60x + 40y = 20$
$\Rightarrow 3x + 2y = 1$ ... (1)

$\Rightarrow 180x + 225y = 2 * 45$
$\Rightarrow 4x + 5y = 2$ ... (2)

From equation (1) and equation (2), we can say that $x = \frac{1}{7}, y = \frac{2}{7}$

It is given that in 6 days 25 workers can manufacture ‘m’ bats and ‘n’ balls. Hence,

$\Rightarrow m * (\frac{1}{7}) + n * (\frac{1}{7}) = 6 * 25$

$\Rightarrow m + 2n = 1050$

We can see that 0 $\leq$ m $\leq$ 1050, 0 $\leq$ n $\leq$ 525.
The value of |m - n| will be minimum when m = n. Therefore,
$\Rightarrow 3n = 1050$
$\Rightarrow n = 350$
Hence, the value of (m+n) = 350 + 350 = 700.

Top-100 CAT Quant Questions Part-2 Video

Question 13

A tank of capacity 1200 litres is being filled by two taps A and B. A can fill the tank in 30 hours and B can fill the tank in 40 hours. Due to extended usage, the tank develops a leak at the bottom. However, due to salt deposition at the site of the leak, the size of the hole keeps decreasing uniformly over time. At the start, water leaks from the hole at the rate of half a liter per minute. After 4 hours, this reduces to 100ml per minute and becomes constant thereafter. If both taps are opened, how long will it take to fill the tank?

A 22.44 hours

B 15.50 hours

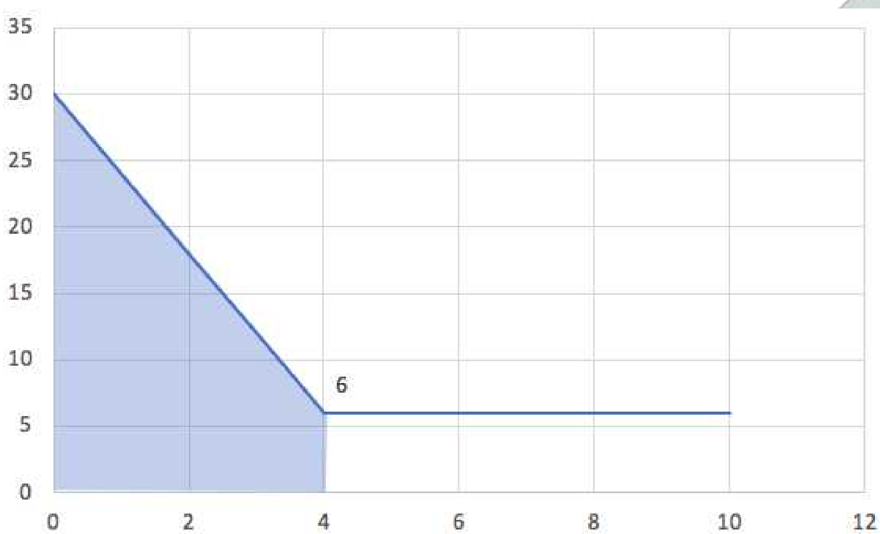
C 18.24 hours

D 19.50 hours

Answer: D

Explanation:
Lets convert leak timings to litres per hour. Initial rate is 0.5 litres per min = 0.5* 60 = 30 lit / hr
Final rate = 0.1*60 = 6 lit / hr

The rate of leaking can be represented by a graph as follows:



The volume of water leaked is given by the area under the graph. In the first four hours, the area is as indicated by the light blue trapezium with sides 30 and 6 and height 4.

Hence, water leaked = 1/2* (30+6) * 4 = 72 litres
Tap A fills the tank at 1200/30 = 40 lit / hr
Tap B fills the tank at 1200/40 = 30 lit / hr

In the first four hours, the tank is filled up to level = 4*(40+30) - 72 = 4*70 - 72 = 208 litres.

After that, rate of filling the tank = 40+ 30 - 6 = 64 lit / hr
Time taken = (1200 - 208) / 64 = 992/64 = 15.5 hours

Thus, total time taken = 15.5 + 4 = 19.5 hours.

Question 14

A tank has two taps attached to it. Tap A fills the empty tank in 10 hours and tap B empties the full tank in 12 hours. At time t = 0, the tank is empty and both the taps are open. Tap A is closed at t = 30 min, reopened at t = 60 min, closed again at t = 90 min and so on. Tap B is closed at t = 45 min, reopened at t = 90 min, closed at t = 135 min and so on. Both the taps are closed as soon as the tank is full. If the tank is full at t = x minutes. Find x.

A 115.5 hours

B 120 hours

C 118 hours 22.5 minutes

D 118.5 hours

Answer: A

Explanation:
Tap A takes 10 hours to fill an empty tank and tap B takes 12 hours to drain a full tank. So, the efficiencies of A and B are in the ratio 12:10. So, A does 12 units of work in 1 hour and B does 10 units of work in 1 hour.
The duration of one cycle (open-close-open) for tap A is 60 minutes and the duration of one cycle (open-close-open) for tap B is 90 minutes.
LCM (60, 90) = 180

Consider the time from t = 0 to t = 180 min:

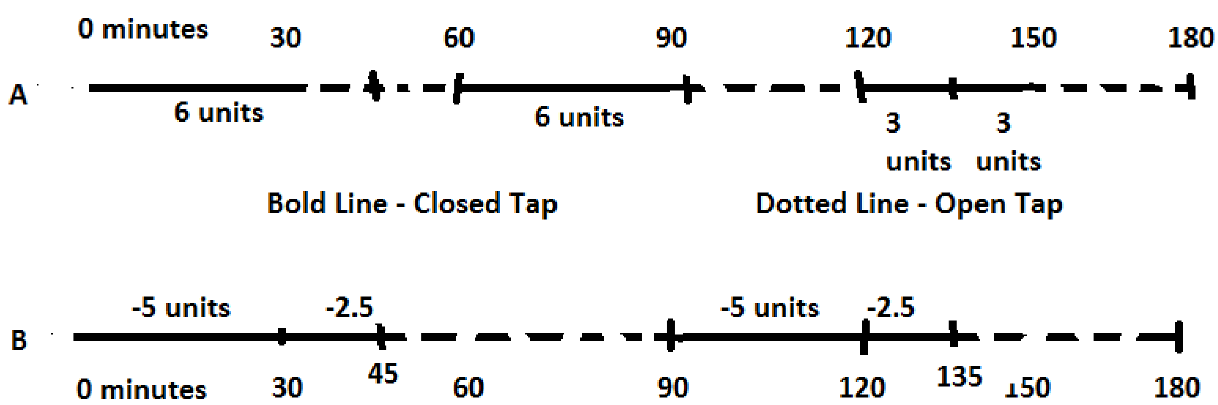
Tap A is open for 90 minutes. So, it does a work of 1.5*12 = 18 units.
Tap B is open for 90 minutes. So, it does a work of 1.5*10 = 15 units.

So, effective work done by both the taps in 180 minutes = 3 units.

Total work needed to be done = 10*12 = 120 units.

So, time required = 120/3 * 180 minutes = 40 * 3 hours = 120 hours.

But, the tank will be full before 120 hours.



Consider the first cycle: Work done in the first half an hour = 1 unit. In every other cycle, work done in the next 15 minutes is -2.5 units by tap B. But, in the first cycle, there is only 1 unit to drain out in the next 15 minutes. So, in effect, there is a gain of 1.5 units of work in the first cycle compared to the other cycles. That is, 4.5 units of work is done in the first cycle.

At the end of the 38th cycle, 115.5 units of work is done. Now, in the 39th cycle, an effective work of 4.5 units is done in the first 90 minutes. This is when 120 units of work is done and the tank is full. Both the taps are now closed.

So, total time required to fill the empty tank = (38*3 + 1.5) hours = 115.5 hours

Question 15

A water tank has inlets of two types A and B. All inlets of type A when open, bring in water at the same rate. All inlets of type B, when open, bring in water at the same rate. The empty tank is completely filled in 30 minutes if 10 inlets of type A and 45 inlets of type B are open, and in 1 hour if 8 inlets of type A and 18 inlets of type B are open. In how many minutes will the empty tank get completely filled if 7 inlets of type A and 27 inlets of type B are open?

Answer:48

Explanation:
Let the efficiency of type A pipe be 'a' and the efficiency of type B be 'b'.
In the first case, 10 type A and 45 type B pipes fill the tank in 30 mins.

So, the capacity of the tank = $\frac{1}{2}$(10a + 45b)........(i)

In the second case, 8 type A and 18 type B pipes fill the tank in 1 hour.
So, the capacity of the tank = (8a + 18b)..........(ii)
Equating (i) and (ii), we get
10a + 45b = 16a + 36b
=>6a = 9b
From (ii), capacity of the tank = (8a + 18b) = (8a + 12a) = 20a
In the third case, 7 type A and 27 type B pipes fill the tank.
Net efficiency = (7a + 27b) = (7a + 18a) = 25a

Time taken = $\frac{20a}{25a}$ hour = 48 minutes.

Hence, 48 is the correct answer.

Question 16

Jyoti and Prakash are running in the same direction starting from the same point on a circular track of length 115 m with speeds 10 m/sec and 2 m/sec respectively. How many times will they be at the maximum possible distance from each other before their third meeting at the starting point?

Answer:12

Explanation:
The ratio of the speeds of Jyoti and Prakash is 5 : 1. So, Jyoti will cover 5 rounds in the time Prakash will cover 1 round. Therefore, they will meet three times before meeting for the fourth time at the starting point. They will be at the maximum possible distance when they will be diametrically opposite to each other. Before each meeting point, they will be diametrically opposite to each other once. So, when they will meet for the first time at the starting point, they would have been diametrically opposite to each other four times. Similarly, when they will meet for the second time at the starting point, they would have been diametrically opposite to each other four more times. Also, before their third meeting at the starting point, they would be diametrically opposite to each other four times.
Required answer = (4 + 4 + 4) = 12
Hence, 12 is the correct answer.

Question 17

A, B and C are climbing up an escalator and for every step C takes, A and B take 2 and 4 steps respectively. If A reaches the top after taking 30 steps and C reaches the top by taking 20 steps, how many steps did B take to reach the top?

A 35

B 40

C 50

D Can't be determined

Answer: B

Explanation:
Suppose the number of steps that need to be taken without the escalator is S.

As the speed of C is half that of A, the number of steps C takes by the time A reaches the top is 15.

While someone is going up an escalator which is also going up, the person covers some steps and the escalator covers some steps. The sum of these both is equal to the total number of steps on the escalator.

Let us say that the escalator covered 'x' steps when A covered 30 steps.

So total number of steps on the escalator = S = 30+x.

As the escalator covers x steps when A takes 30 steps,it must cover x steps when C takes 15 steps and it must cover 4x/3 steps when C takes 20 steps.

So, 30+x = 20+4x/3

=> x = 30.

Thus the total number of steps on the escalator is 60.

When A takes 30 steps the escalator also takes 30 steps. This means that the speed of the escalator is the same as the speed of A.

But the speed of B is twice that of A.

So B must take twice the number of steps as the escalator to go to the top.

So, 2y+y=60 => y=20 => 2y = 40.

Thus, number of steps B took to reach the top is 40.

Question 18

Ratnesh used an escalator often in a shopping complex. He noticed that if he walked 30 steps on the escalator, he needs 42 seconds to reach to bottom. However, if he walked quickly and completed 45 steps on the same escalator he needed only 24 seconds to reach to the bottom. If escalator is moving downwards at a constant speed, what is the number of steps on the escalator ?

Answer:65

Explanation:
Let us assume that in first case escalator moves by x steps in 42 seconds.

In second case escalator is in function for only 24 seconds hence steps covered by the escalator in 24 seconds = $\frac{x}{42} \times 24 = \frac{4x}{7}$

We can see that $30 + x = 45 + \frac{4x}{7}$

On solving for x, x = 35

Hence total number of steps in escalator = 30 + x = 30 + 35 = 65 steps.

CAT Formulas PDF [Download Now]

Question 19

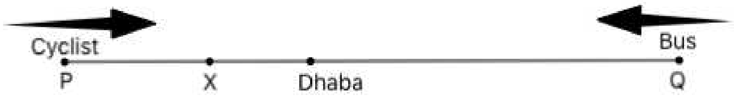
A cyclist and a bus start from cities P and Q respectively at the same time. Every day the cyclist and the bus cross each other at a road-side Dhaba. One day the cyclist started 42 minutes later than his usual time. As a result, the bus and the cyclist meet at a point which is 5km away from the restaurant. Find out the speed(in km/h) of the bus, if the cyclist cycled at a uniform speed of 10 km/h.

Answer:25

Explanation:

The above figure represents the movement of the cyclist and the bus between cities P and Q.
As the cyclist started 42 minutes later than his usual time but the bus started at its usual time. So, when the bus would have reached the Dhaba (their usual meeting point), the cyclist would be 42 minutes away from the Dhaba.

Distance covered by cyclist in 42 minutes = $\frac{42}{60}$ hr $\times 10$ km/hr = 7 km



Let, their new meeting point be somewhere between X and Dhaba. The cyclist has reached point X when Bus has reached the Dhaba. The distance between X and Dhaba is 7km. Since, they meet at a point 5 km away from dhaba. So, the cyclist must have cycled for 2 Km which would have taken him 12 minutes (= $\frac{2}{10} \times 60$).

So, the bus would also have travelled 5km in 12 minutes after crossing the dhaba to meet the cyclist.

Speed of bus = $\frac{5}{\frac{12}{60}}$ km/hr = 25 km/hr.

Hence, the correct answer is 25 km/hr.

Question 20

A rectangular tank of height 20 m is fitted with an inlet pipe that can fill the whole tank in 60 minutes. Three holes of different sizes are being made in the tank at the heights of 15m, 10m, and 5m such that if they are made at the bottom, they can empty the whole tank in 360 minutes, 180 minutes, and 240 minutes respectively. How much time will it take for the tank to get 80% filled? (Assume that the rate of flow depends only on the area of hole and not on the height at which the hole is present)

A 83 minutes

B 131 minutes

C 116 minutes

D 96 minutes

Answer: A

Explanation:
Assume the total tank to be of 720 units.
So, Inlet pipe can fill 720/60=12 units/min
15 m height hole can empty 720/360=2 units/min
10 m height hole can empty 720/180=4 units/min
And 5 m height hole can empty 720/240=3 units/min.
Each of the pipes will be non-functional until the water level reaches their respective heights.

So until 5m, only inlet pipe is functional. Time taken will be $\frac{720}{12\times4} = 15$ minutes

From 5-10 m, 5m height hole will leak water at 3 units/min. So, water gets filled at the rate of 9 units/min. Time taken will be $\frac{720}{9\times4} = 20$ minutes

From 10-15, 10m height hole will also become functional. Thus, water gets filled at a rate of 5 units/min. Time taken will be $\frac{720}{5\times4} = 36$ minutes

80% of 20m is 16m. So, for 15m-16m, all the pipes will be functional. So, water rate will be 3 units/min. Time taken will be $\frac{720}{3\times20} = 12$ minutes
=> Total time taken is 15+20+36+12=83 minutes.

Question 21

A jailor was checking the honesty of 3 prisoners who were imprisoned in cells 4, 5, and 6. He gathered the three prisoners and told them that they will be asked to take sweets from a sweet box in a separate room in the ratio of their respective cell numbers. They were called in the descending order of their cell numbers. However, the prisoners from cell numbers 4 and 5 took sweets thinking that they were the first one to enter the room. The prisoner from cell number 6, however, thought that he was the second one to enter the room after inmate from cell 4. What proportion of sweets were remaining at the end?

A $\frac{8}{99}$

B $\frac{13}{66}$

C $\frac{2}{9}$

D None of these

Answer: C

Explanation:
So, they are asked to take sweets in the ratio 4:5:6
Assume that there are 'x' sweets.
First to enter is inmate from cell 6. He thinks that the prisoner from cell 4 has already taken sweets. So, he will take the sweets in the ratio 6:5.
=> He must take 6x/11 sweets leaving 5x/11 behind.
Now enters prisoner from cell 5 who thinks he is the first one to enter. So, he will take 5/15 or 1/3rd of the sweets leaving 2/3rd of the sweets already present.
So, the sweets remaining will be 2/3 X 5x/11 = 10x/33
Now enters prisoner from cell 4. He will take 4/15th of the sweets remaining leaving 11/15th of sweets present.
So, finally the sweets remaining are 10x/33 X 11/15 =2x/9
Thus, C is the correct answer.

CAT Previous Papers PDF

Question 22

The strength of a salt solution is p% if 100 ml of the solution contains p grams of salt. If three salt solutions A, B, C are mixed in the proportion 1 : 2 : 3, then the resulting solution has strength 20%. If instead the proportion is 3 : 2 : 1, then the resulting solution has strength 30%. A fourth solution, D, is produced by mixing B and C in the ratio 2 : 7. The ratio of the strength of D to that of A is

A 3 : 10

B 1 : 3

C 1 : 4

D 2 : 5

Answer: B

Explanation:
Let 'a', 'b' and 'c' be the concentration of salt in solutions A, B and C respectively.

It is given that three salt solutions A, B, C are mixed in the proportion 1 : 2 : 3, then the resulting solution has strength 20%.

$$\Rightarrow \frac{a+2b+3c}{1+2+3} = 20$$

$\Rightarrow a+2b+3c = 120$ ... (1)

If instead the proportion is 3 : 2 : 1, then the resulting solution has strength 30%.

$$\Rightarrow \frac{3a+2b+c}{1+2+3} = 30$$

$\Rightarrow 3a+2b+c = 180$ ... (2)

From equation (1) and (2), we can say that

$\Rightarrow b+2c = 45$

$\Rightarrow b = 45-2c$

Also, on subtracting (1) from (2), we get

$a-c = 30$

$\Rightarrow a = 30+c$

In solution D, B and C are mixed in the ratio 2 : 7

So, the concentration of salt in D = $\frac{2b+7c}{9} = \frac{90-4c+7c}{9} = \frac{90+3c}{9}$

Required ratio = $\frac{\frac{90+3c}{9a}}{\frac{90+3c}{9(30+c)}}$ = 1 : 3

Hence, option B is the correct answer.

Question 23

Amala, Bina, and Gouri invest money in the ratio 3 : 4 : 5 in fixed deposits having respective annual interest rates in the ratio 6 : 5 : 4. What is their total interest income (in Rs) after a year, if Bina's interest income exceeds Amala's by Rs 250?

A 6350

B 6000

C 7000

D 7250

Answer: D

Explanation:
Assuming the investment of Amala, Bina, and Gouri be 300x, 400x and 500x, hence the interest incomes will be 300x*6/100=18x, 400x*5/100=20x and 500x*4/100 = 20x

Given, Bina's interest income exceeds Amala by 20x-18x=2x=250 => x=125

Now, total interest income = 18x+20x+20x=58x = 58*125 = 7250

Question 24

Ravi appeared in 5 exams - English, Maths, Science, Social science and Computer science. The total marks allotted for each subject is different and is a multiple of 50. 40% of the total marks allotted to a subject is considered pass marks. The total marks allotted for the 5 subjects put together is 750. Ravi passed in each subject. The difference between his marks in English and the pass mark in English is 10% of the pass marks. For Maths, this difference is 20% of the pass marks, 30% for Science, 40% for Social science and 50% for computer science. If Ravi obtained 410 marks in total, the difference between the marks obtained by him in Maths and Social Science is

A 64

B 56

C 34

D Cannot be determined

Answer: A

Explanation:
It has been given that the total marks in each subject is different and is a multiple of 50. Let the total marks in English be 50a, Maths be 50b, Science be 50c, Social science be 50d and Computer science be 50e, where a,b,c,d, and e are different integers. Total marks allotted for the 5 subjects = 750

=> 50a + 50b + 50c + 50d + 50e = 750

a + b + c + d + e = 15.

The least value that 'a' can take is 1.

1 + 2 + 3 + 4 + 5 = 15.
Therefore, the marks allotted to the 5 subjects should be 50 , 100, 150, 200, and 250.

40% of the total marks allotted to a subject is pass marks. Therefore, the pass marks in the 5 subjects are 20, 40, 60, 80, and 100. It has been given that Ravi obtained 10% more than the pass marks in English, 20% more than the pass marks on Maths, 30% more than the pass marks in Science, 40% more than the pass marks in Social science and 50% more than the pass marks in computer science. Had Ravi obtained the pass marks in all the subjects, he would have received 20 + 40 + 60 + 80 + 100 = 300 marks. We know that Ravi has obtained 410 marks in total or 110/300 = 11/30 more than the pass marks.

The pass marks are in the ratio 1:2:3:4:5. Let the pass marks be x, 2x , 3x, 4x and 5x. Had Ravi obtained the pass marks, he would have obtained 15x. However, Ravi has obtained 15x + 15x*11/30 = 20.5x in total.

We have to multiply 1.1, 1.2, 1.3, 1.4 and 1.5 with x, 2x, 3x, 4x, and 5x such that the sum of the values obtained is 20.5x. Let us find out the least possible and maximum possible values that can be obtained using the combinations.

The least possible value will occur when 1.1 is multiplied with 5x, 1.2 is multiplied with 4x, 1.3 is multiplied with 3x, 1.4 is multiplied with 4x and 1.5 is multiplied with 5x. In this case, the total marks obtained by Ravi will be 1.1*5x + 1.2*4x + 1.3*3x + 1.4*2x + 1.5*x = 5.5x + 4.8x + 3.9x + 2.8x + 1.5x = 18.5x.

The maximum possible value will occur when 1.1*x + 1.2*2x + 1.3*3x + 1.4*4x + 1.5*5x = 1.1x + 2.4x + 3.9x + 5.6x + 7.5x = 20.5x.

The given case represents the maximum possible score that could have been scored by Ravi for the given conditions. Therefore, the English exam must be of 50 marks, Maths exam must be of 100 marks, Science exam must be of 150 marks, Social science exam must be of 200 marks and Computer science exam must be of 250 marks.

Difference between the marks obtained by Ravi in Maths and Social Science = 200*0.4*1.4 - 100*0.4*1.2 = 112 - 48 = 64.

Therefore, option A is the right answer.

Complete Quant in 4 Hours (Revision Video)

Question 25

John and Andrew have a certain number of identical chocolates with them. John gives five-eighths of the chocolates with him to Andrew. After receiving the chocolates, Andrew gives three-eighths of the chocolates with him to John. A transfer cycle is said to be completed when John receives chocolates back from Andrew. After every transfers cycle, the ratio of the number of chocolates with John and Andrew remains constant. If Andrew had 8 more chocolates than John initially, then what is the total number of chocolates that John and Andrew had together initially?

Answer:392

Explanation:
Let the number of chocolates initially with John be J and Andrew be A.
After every transfers cycle, the ratio of the number of chocolates with John and Andrew remains constant. Therefore, after 1 round of exchange, the ratio of the number of chocolates with them should be equal to the ratio of number of chocolates they had initially.

Number of chocolates with John initially = J.
After John gives 5/8 th of the number of chocolates with him, John will have J - 5J/8 = 3J/8 chocolates.
Andrew will receive 5J/8 chocolates from John. Therefore, the number of chocolates with Andrew will be A + 5J/8.

Now, Andrew gives John $3/8^{th}$ of the total number of chocolates with him.
Therefore, Andrew will give John 3A/8 + 15J/64 chocolates.
John, after receiving the chocolates from Andrew, will have 3J/8 + 3A/8 + 15J/64 chocolates.
We know that after one complete round of exchange, the ratio of number of chocolates with them should be equal to the ratio of number of chocolates they had initially.

$$\Rightarrow \frac{39J/64 + 3A/8}{5A/8 + 25J/64} = \frac{J}{A}$$

$\Rightarrow 39AJ + 24A^2 = 40AJ + 25J^2$
$\Rightarrow 24A^2 - AJ - 25J^2 = 0$
$\Rightarrow (24A - 25J)(A + J) = 0$
$\Rightarrow A \neq J$. Hence, $24A = 25J$

Also, it has been given in the question that Andrew had 8 more chocolates than John initially.
$\Rightarrow A = J + 8$
$\Rightarrow 24(J + 8) = 25J$
$\Rightarrow J = 192.$

Therefore, the total number of chocolates with John and Andrew initially = A + J = (J + 8) + J = (192+8) + 192 = 392.

Question 26

A class consists of 20 boys and 30 girls. In the mid-semester examination, the average score of the girls was 5 higher than that of the boys. In the final exam, however, the average score of the girls dropped by 3 while the average score of the entire class increased by 2. The increase in the average score of the boys is

A 9.5

B 10

C 4.5

D 6

Answer: A

Explanation:
Let, the average score of boys in the mid semester exam is A.
Therefore, the average score of girls in the mid semester exam be A+5.
Hence, the total marks scored by the class is $20 \times (A) + 30 \times (A+5) = 50 \times A + 150$

The average score of the entire class is $\frac{(50 \times A + 150)}{50} = A + 3$

wkt, class average increased by 2, class average in final term $= (A+3) + 2 = A + 5$

Given, that score of girls dropped by 3, i.e $(A+5) - 3 = A + 2$
Total score of girls in final term $= 30 \times (A+2) = 30A + 60$

	MID-TERM		FINAL-TERM	
No Boys/Girls	Avg score	Total score	Avg score	Total score
20	A	20A		
30	A+5	30A+150	A+2	30A+60
Class score		50A+150		
Class Average		A+3		A+5

Total class score in final term $= (A+5) \times 50 = 50A + 250$

the total marks scored by the boys is $(50A + 250) - (30A - 60) = 20A + 190$

Hence, the average of the boys in the final exam is $\frac{(20G + 190)}{20} = A + 9.5$

	MID-TERM		FINAL-TERM	
No Boys/Girls	Avg score	Total score	Avg score	Total score
20	A	20A	A+9.5	20A+190
30	A+5	30A+150	A+2	30A+60
Class score		50A+150		50A+250
Class Average		A+3		A+5

Hence, the increase in the average marks of the boys is $(A + 9.5) - A = 9.5$

Question 27

There are two drums, each containing a mixture of paints A and B. In drum 1, A and B are in the ratio 18 : 7. The mixtures from drums 1 and 2 are mixed in the ratio 3 : 4 and in this final mixture, A and B are in the ratio 13 : 7. In drum 2, then A and B were in the ratio

A 251 : 163

B 239 : 161

C 220 : 149

D 229 : 141

Answer: B

Explanation:
It is given that in drum 1, A and B are in the ratio 18 : 7.

Let us assume that in drum 2, A and B are in the ratio x : 1.

It is given that drums 1 and 2 are mixed in the ratio 3 : 4 and in this final mixture, A and B are in the ratio 13 : 7.

By equating concentration of A

$$\Rightarrow \frac{3 * \frac{18}{18+7} + 4 * \frac{x}{x+1}}{3+4} = \frac{13}{13+7}$$

$$\Rightarrow \frac{54}{25} + \frac{4x}{x+1} = \frac{91}{20}$$

$$\Rightarrow \frac{4x}{x+1} = \frac{239}{100}$$

$$\Rightarrow x = \frac{239}{161}$$

Therefore, we can say that in drum 2, A and B are in the ratio $\frac{239}{161}$: 1 or 239 : 161.

Question 28

Four unique even numbers are arranged in ascending order and the first two are decreased by 50%, while the last two are increased by 50%. If the original sum of the 4 numbers was 162, the new sum can be?

A 165

B 164

C 173

D 160

Answer: C

Explanation:
Let the numbers be 2a, 2b, 2c, 2d.

and $a \leq b \leq c \leq d$

and at first, 2a + 2b + 2c + 2d = 162

which means a + b + c + d =81 --------equation (1)

and we have to find the value of a + b + 3c + 3d

which is (a + b + c + d) + 2c + 2d ; putting values from equation (1)

= 81 + 2c +2d

= 81 + 2 (c + d) ---------equation(2)

also if $a \leq b \leq c \leq d$

$then\ a + 1 \leq b$

$b + 1 \leq c$

$c + 1 \leq d$

from these equations :

$a + 2 \leq c$ ---------------equation(3)

$b + 2 \leq d$ ---------------equation(4)

adding equation (3) and (4)

$a + b \leq c + d - 4$ ---------- equation(5) putting values of a+b from equation (5) to equation (1)

$c + d - 4 + c + d \geq 81$

$2(c + d) \geq 85$

$c + d \geq 42.5$

as c and d are natural numbers so $c + d \geq 43$

putting this value in equation (2)

a+b+3c+3d $\geq$ 81 + 2*(43)

a+b+3c+3d $\geq$ 167

which means the minimum value of the required equation is 167.

We only have one option greater than 167 which is 173.

Hence 173 is the answer.

Question 29

A CAT aspirant appears for a certain number of tests. His average score increases by 1 if the first 10 tests are not considered, and decreases by 1 if the last 10 tests are not considered. If his average scores for the first 10 and the last 10 tests are 20 and 30, respectively, then the total number of tests taken by him is

Answer:60

Explanation:
Let the total number of tests be 'n' and the average by 'A'
Total score = n*A
When 1st 10 tests are excluded, decrease in total value of scores = (nA - 20 * 10) = (nA - 200)
Also, (n - 10)(A + 1) = (nA - 200)
On solving, we get 10A - n = 190..........(i)
When last 10 tests are excluded, decrease in total value of scores = (nA - 30 * 10) = (nA - 300)
Also, (n - 10)(A - 1) = (nA - 300)
On solving, we get 10A + n = 310..........(ii)
From (i) and (ii), we get n = 60
Hence, 60 is the correct answer.

Question 30

A solution contains alcohol and water in the ratio 3 : 2. Sumit replaced some quantity of the solution with water such that the ratio of alcohol and water was reversed. What percentage of the solution was replaced? (Round off the answer to the nearest integer)

Answer:33

Explanation:
We can use the formula

Final proportion = Initial Proportion (1 - $\frac{QuantityReplaced}{TotalQuantity}$)

or, $\frac{2}{5}$ = $\frac{3}{5}$(1 - $\frac{QuantityReplaced}{TotalQuantity}$)

or, $\frac{QuantityReplaced}{TotalQuantity}$ = 1- $\frac{2}{3}$

or, $\frac{QuantityReplaced}{TotalQuantity}$ = $\frac{1}{3}$ = 33%

Hence, the correct answer is 33

Question 31

The cost of 4 pens, 10 pencils and 14 erasers at a stationery shop is Rs 120. At the same shop, the cost of 6 pens and 10 erasers is Rs 60 more than 12 pencils. By how much amount (in Rs.) does the cost of 78 pencils and 2 erasers exceed the cost of 12 pens?

Answer:120

Explanation:
Let the cost of a pen be x, the cost of a pencil be y and the cost of an eraser be z.
4x + 10y + 14z = 120
=> 2x + 5y + 7z = 60 → (1)
6x + 10z = 60 + 12y
=> 6x - 12y + 10z = 60
=> 3x - 6y + 5z = 30 → (2)
We need the value of 78y + 2z - 12x = 2*(-6x + 39y + z)
3*(1) - 4*(2) gives 6x + 15y + 21z - 12x + 24y - 20z = 180 - 120
=> -6x + 39y + z = 60
So, the required value is 2*60 = 120

Question 32

Ravi has coins of denominations Rs 5 and Rs 16 only. Find the sum of digits of the maximum whole number amount that he cannot pay using these denominations and also the number of such amounts that he cannot pay using the coins he has.

A 12, 25

B 14, 30

C 12, 30

D 14, 25

Answer: B

Explanation:
The problem can be re-written as "find the largest whole number that cannot be represented by the linear equation 5x + 16y such that both x and y are non-negative integers".
Here, let's use the Chicken McNugget's Theorem which states that if a and b are positive co-prime integers, then the largest positive integer that cannot be represented by ma + nb is (ab - a - b). The number of such positive integers that cannot be represented by ma + nb is equal to $\frac{(a-1)(b-1)}{2}$.
5x + 16y => Largest amount that cannot be represented by these denominations = (16)(5) - 16 - 5 = 59 => Sum of the digits = 5 + 9 = 14
=> Number of amounts that cannot be represented by these denominations = $\frac{(16-1)(5-1)}{2}$ = 30.
Hence B is the answer.

Alternate Method:
16 = 1(Mod 5) => Any number greater than 16 and of the form 5k+1 can be represented by 5x + 16y
32 = 2(Mod 5) => Any number greater than 32 and of the form 5k+2 can be represented by 5x + 16y
48 = 3(Mod 5) => Any number greater than 48 and of the form 5k+3 can be represented by 5x+16y
64 = 4(Mod 5) => Any number greater than 64 and of the form 5k+4 can be represented by 5x+16y
=> Last 5k+4 number less than 64 cannot be represented by 5x+16y => 59 is the answer.

The number of numbers of the form 5k+1 which cannot be represented is 1, 6, 11. => 3
The number of numbers of the form 5k+2 which cannot be represented is 2, 7, 12 ... 27 => 6
The number of numbers of the form 5k+3 which cannot be represented is 3, 8, 13, ... 43 => 9
The number of numbers of the form 5k+4 which cannot be represented is 4, 9, 14 . . 59 => 12
Therefore, the number of such numbers = 3 + 6 + 9 + 12 = 30

Question 33

5 Cabbages, 12 Coconuts and 19 Brinjals cost Rs. 275. 23 Cabbages, 19 Coconutes and 15 Brinjals cost Rs.360. How much do 1 Coconut, 1 Brinjal and 1 Cabbage cost?

A 17

B 18

C 20

D Can't be determined

Answer: C

Explanation:
5X+12Y+19Z=275 -> 1st Equation
23X+19Y+15Z = 360 ->2nd Equation.

Note that the difference in the coefficients of the first equation is 7 while the difference in the coefficients of the second equation is 4. So, 4* First Equation + 7 * Second Equation, gives X+Y+Z =20

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Question 34

In a football team having 'n' teams each, all the teams play each other twice and get 3 points for a win and 1 point each for a draw. If the total number of points earned by the teams is 217, how many games were drawn?

A 37

B 53

C 71

D Can’t be determined

Answer: B

Explanation:
Let x represent the number of wins and y represent the number of draws.

For every win, 3 point will be added to 1 team and 0 to other. Hence, net increase in points = 3x.

For every draw, 1 point will be added to both the teams. So, net increase = 2y.
3x+2y = 217 and x+y = n*(n-1) for some n. The only integral solutions to the above equation are x=37 and y = 53 and n=10

Question 35

For any two positive integer 'a' and 'b', what is the product of all possible values of 'a' for which 1/a + 1/b = 2/9 and a<b

A 24

B 30

C 12

D 48

Answer: B

Explanation:
1/a + 1/b = 2/9

Taking LCM, we get (a+b)/ab = 2/9

Cross multiplying, we get 9a+9b = 2ab

9a = 2ab-9b

9a = b(2a-9)

b = 9a/(2a-9). The only solutions for (a,b) are {(5,45) and (6,18)}

Question 36

The cost of 5 apples, 7 bananas and 9 mangoes is Rs 204 and the cost of 8 apples, 5 bananas and 2 mangoes is Rs 159. Find the total cost of 1 apple, 1 banana and 1 mango.

Answer:30

Explanation:
Let the cost of 1 apple, 1 banana and 1 mango be a, b and c respectively.
5a + 7b + 9c = 204
8a + 5b + 2c = 159
Coefficients are in AP in both the equations.
The differences of coefficients in the first equation are 2 and the differences of coefficients in the second equation are 3.
Hence, multiply the first equation by 3 and multiply the second equation by 2.
15a + 21b + 27c = 612
16a + 10b + 4c = 318
=> 31a + 31b + 31c = 930
=> a + b + c = 30

Question 37

A survey was conducted in a school to find the inclination of the students to read a subject. It was found that 89% students liked Maths, 92% students liked Physics, 85% students liked Chemistry, 74% students liked Biology, 78% students liked Languages and 2% students liked none of the above-mentioned subjects. If the total number of students in the school is 400, then the minimum number of students who liked all the five subjects is
(Enter '-1' if your answer is Cannot be determined)

Answer:104

Explanation:
The percentage of students in school=100%
Percentage of students who like none of the above mentioned subjects=2%

Consider the number of students who like exactly 1 subject = I

The number of students who like exactly 2 subjects = II

The number of students who like exactly 3 subjects = III

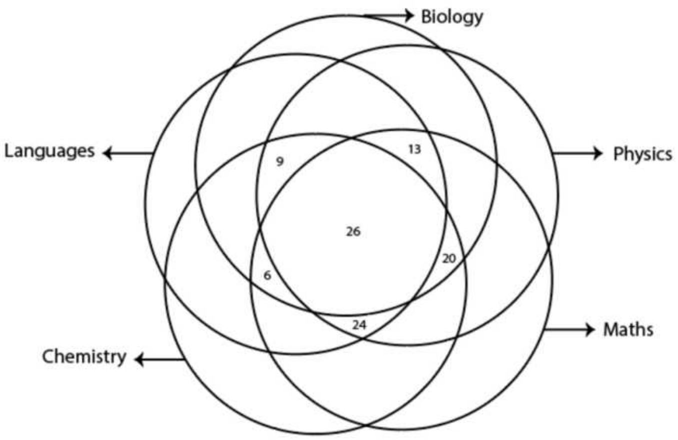
The number of students who like exactly 4 subjects = IV

The number of students who like exactly 5 subjects = V

I+II+III+IV+V=98
I+2II+3III+4IV+5V=89+92+85+74+78=418
Since we have to find the minimum number of students who like all the five subjects, we have to maximize the number of students who like four subjects.
So the rest of the variables can be zero.
4IV+5V=418
IV+V=98
V=26% and IV = 72%

Here we got the minimum percentage of students who like all the five subjects.

In this case, 72% of the students like exactly 4 subjects each and 26% students like all the five subjects. The venn diagram for such a case is given below



To find the number of students, we multiply it with the total number of students and hence it equals = 400*26/100 = 104

Question 38

Out of 400 students of an Engineering college, 220 students are giving GRE, 300 students are giving CAT and 50 students are giving GMAT. If it is known that all the students are taking atleast one exam, what is the maximum possible number of students who are giving all three exams?

A 45

B 85

C 110

D 50

Answer: D

Explanation:
Let R, M and C be the sets of students of GRE, GMAT and CAT. Let x be the # who are taking all three exams.
To maximize x, we assume that $n(R \cap M \cap C') = n(R \cap C \cap M') = n(M \cap C \cap R') = 0.$
Hence, 400=(n(R)-x)+(n(M)-x)+(n(C)-x)+x=300+220+50-2x=400.
Hence x=85. However, x cannot be greater than 50. Hence the maximum # of students is 50.

Question 39

In a college, four-fifths of the students own a desktop and half of the students own a laptop. 40% of the students own both. What is the ratio of the number of students who own exactly one of desktop and laptop and the number of students who own neither a desktop nor a laptop?

A 5 : 1

B 1 : 5

C 4 : 1

D 1 : 4

Answer: A

Explanation:
Let the total number of students in the college be N.
Number of students who own a desktop = 4N/5
Number of students who own a laptop = N/2
Number of students who own both = 2N/5
So, number of students who own only desktop = 4N/5 - 2N/5 = 2N/5
Number of students who own only laptop = N/2 - 2N/5 = N/10
So, number of people who own only desktop or only laptop = 2N/5 + N/10 = N/2
Number of people who do not own either a desktop or a laptop = N - N/2 - 2N/5 = N/10
Required ratio = N/2 : N/10 = 5 : 1

Question 40

Shyam visited Ram during his brief vacation. In the mornings they both would go for yoga. In the evenings they would play tennis. To have more fun, they indulge only in one activity per day, i.e. either they went for yoga or played tennis each day. There were days when they were lazy and stayed home all day long. There were 24 mornings when they did nothing, 14 evenings when they stayed at home, and a total of 22 days when they did yoga or played tennis. For how many days Shyam stayed with Ram?

A 32

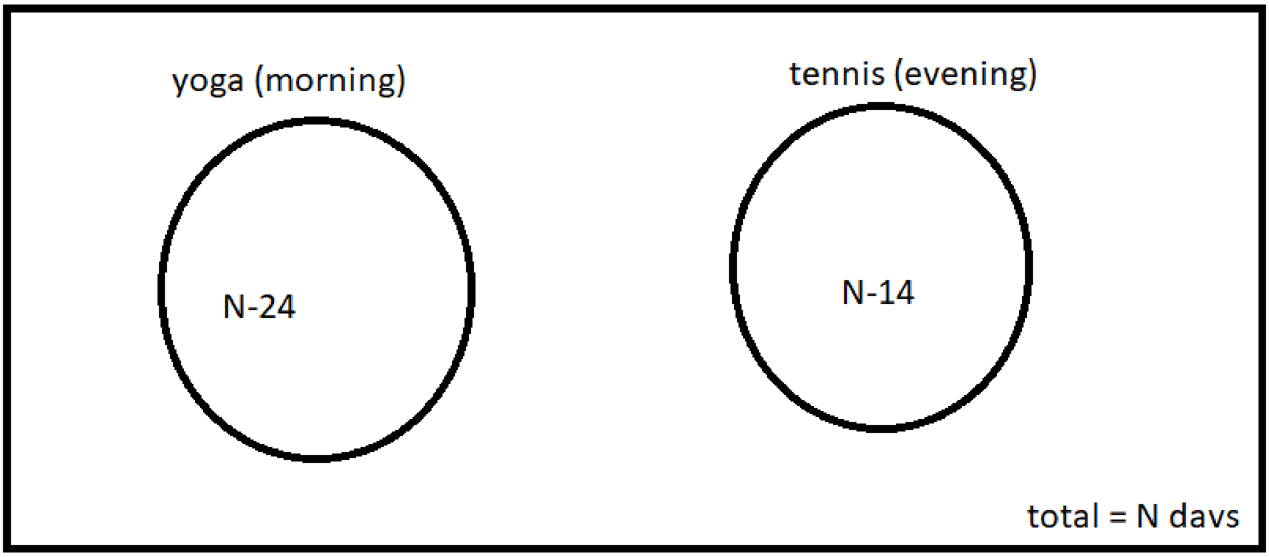
B 24

C 30

D None of these

Answer: C

Explanation:
Let the number of total days=N
They played tennis for=N-14 days
They did yoga for =N-24 days



And the question says that total days when they did yoga or played tennis are 22

which means

N-14 + N-24 = 22

2N - 38 = 22

2N = 60

N = 30

Hence total days they stayed together were 30

Question 41

Let $a_1, a_2 \ldots a_{52}$ be positive integers such that $a_1 < a_2 < \ldots < a_{52}$. Suppose, their arithmetic mean is one less than arithmetic mean of $a_2, a_3, \ldots a_{52}$. If a_{52}= 100, then the largest possible value of a_1 is

A 48

B 20

C 23

D 45

Answer: C

Explanation:
Let 'x' be the average of all 52 positive integers $a_1, a_2 ... a_{52}$.

$a_1 + a_2 + a_3 + ... + a_{52}$ = 52x ... (1)

Therefore, average of a_2, a_3, ...a_{52} = x+1

$a_2 + a_3 + a_4 + ... + a_{52}$ = 51(x+1) ... (2)

From equation (1) and (2), we can say that

$a_1 + 51(x + 1)$ = 52x

a_1 = x - 51.

We have to find out the largest possible value of a_1. a_1 will be maximum when 'x' is maximum.

(x+1) is the average of terms a_2, a_3, ...a_{52}. We know that a_2 < a_3 < ... < a_{52} and a_{52} = 100.

Therefore, (x+1) will be maximum when each term is maximum possible. If a_{52} = 100, then a_{52} = 99, a_{50} = 98 ends so on.

a_2 = 100 + (51-1)*(-1) = 50.

Hence, $a_2 + a_3 + a_4 + ... + a_{52}$ = 50+51+...+99+100 = 51(x+1)

$$\Rightarrow \frac{51 * (50 + 100)}{2} = 51(x + 1)$$

$$\Rightarrow x = 74$$

Therefore, the largest possible value of a_1 = x - 51 = 74 - 51 = 23.

Question 42

The average of 33 consecutive 3 digit even numbers increases by 6 if the digits of 30th number in series are reversed. If digits of the 30th term are in strictly increasing or strictly decreasing order and hundreds digits of all numbers are the same, what is the sum of digits of the 3rd term in series?

A 15

B 12

C 21

D 18

Answer: B

Explanation:
Consider average of given numbers = a

Total sum = 33a

Assuming x,y and z are hundreds digit, tens digit and units digit of 30th term of the series respectively.

After reversing the digits, new average = $\frac{33a-(100x+10y+z)+((100z+10y+z))}{33}$ = $a + 6$

⇒ a + $\frac{99(z-x)}{33}$ = $a + 6$

⇒ $z - x$ = 2

x cannot be 0 as all are 3 digit numbers

Case 1: z= 8 x= 6 y=7 The digits are in strictly increasing or strictly decreasing order.

30th number = 678 =a+2*29 Hence 1st term of series = 620, Last term = a+32*d = 684

Hundreds digits of all numbers are the same.

Case 2: z= 6 x= 4 y=5 The digits are in strictly increasing or strictly decreasing order.

30th number = 456 =a+2*29 Hence 1st term of series = 398, Last term = a+32*d = 462

Hundreds digits of all numbers are not the same.

Case 3: z= 4 x= 2 y=3 The digits are in strictly increasing or strictly decreasing order.

30th number = 234 =a+2*29 Hence 1st term of series = 176, Last term = a+32*d = 240

Hundreds digits of all numbers are not the same.

Only 1st case is possible.

3rd term = a+2*2 = 624

Sum of digits of 624 = 6+2+4 = 12

Question 43

What is the value of the following expression?

$(1/(2^2-1))+(1/(4^2-1))+(1/(6^2-1))+...+(1/(20^2-1)$

A 9/19

B 10/19

C 10/21

D 11/21

Answer: C

Explanation:

$(1/(2^2-1))+(1/(4^2-1))+(1/(6^2-1))+...+(1/(20^2-1)$ = 1/[(2+1)*(2-1)] + 1/[(4+1)*(4-1)] + ... + 1/[(20+1)*(20-1)]

= 1/(1*3) + 1/(3*5) + 1/(5*7) + ... + 1/(19*21)

=1/2 * (1/1 - 1/3 + 1/3 - 1/5 + 1/5 - 1/7 + ... +1/19 - 1/21)

=1/2 * (1 - 1/21) = 10/21

Question 44

Consider an AP consisting of 1000 terms with the sum of those terms as 2000000 and the first term as 2. A new ascending AP with 20 terms is considered from within this AP, such that the sum of its terms is 8000. Find the number of such new AP's that can be formed.

A 10

B 15

C 5

D 0

Answer: C

Explanation:

In the original AP:

n/2[2a + (n-1)d] = 2000000

500 [4+999d]=2000000

4+999d = 4000

d=4.

The original AP is 2,6,10,.....3998. This AP is the form 4k-2 where $1 \leq k \leq 1000$

The sum of all terms of the new AP is of the form n/2[2a+(n-1)d]=8000.

Here a must be of the form (4k-2) and d must be of the form 4l.

The sum becomes 20/2 [2(4k-2) + 19 (4l)] = 8000

8k + 76l =804

2k +19l =201

We find that l must be an odd number to satisfy the equation as the sum is an odd number.

We find that it is satisfied for l = 1,3,5,7,9. When l = 11, k becomes -4 which is not possible as k is defined only from $1 \leq k \leq 1000$

Therefore there are 5 new AP's that can be formed.

Question 45

The mean and median of six positive integers, of which at least 5 are different, are 17 and 22 respectively. What could be the maximum value of the largest number?

A 32

B 33

C 34

D 31

Answer: A

Explanation:
Let the six numbers be a, b, c, d, e and f arranged in the ascending order such that f is the largest number.
Since the mean is equal to 17, the sum of the six numbers will be 17 * 6 = 102
Also, the median is equal to 22. So, the average of the c and d will be 22. Also, the sum of c and d will be 22 * 2 = 44. So, c can be 21 and d can be 23 or c and d both can be 22 each.

When c = 21 and d = 23
To maximize f, we have to minimize a and b. So, we will let a = 1 and b = 1
Also, e must be greater than d as there must be at least 5 different numbers. So, let e = 24
Therefore, f = 102 - (1 + 1 + 21 + 23 + 24) = 32

When c = d = 22
To maximize f, we have to minimize a and b. So, we will let a = 1 and b = 2
Also, e must be greater than d as there must be at least 5 different numbers. So, let e = 23
Therefore, f = 102 - (1 + 2 + 22 + 22 + 23) = 32

Hence, option A is the correct answer.

Alternate Solution:

Mean = 17, hence the sum =6*17=102

Now, a+b+c+d+e+f=102, where a,b,c,d,e,f are in ascending order.

Now to maximize f which is the largest number, we will take a=b=1

c+d=22*2=44, Now the sum e+f=102-(1+1+44)=56

To maximize f, e should be minimized.

c+d=44, since c can not be equal to d, we will take c=21, d=23, (since c<d)

The minimum value of e will be 24. (e>d) =>f=56-24=32......(1)

If we take c=20, d=24, the minimum value of e will be 25. So f= 56-25=31

Hence similarly we can reject c=19, d=25 and so on because the value of f will get smaller.

The largest value of f is 32.

Question 46

If $T_n = 1 - \frac{3}{n}$, then find out the value of $\frac{\prod_{n=51}^{n=100} T_n}{\prod_{n=4}^{n=50} T_n}$, where $\prod_{n=1}^{n=r} T_n = T_1 * T_2 * T_3 * ... * T_r$.

A $\frac{9800}{33}$

B $\frac{19600}{33}$

C $\frac{39200}{33}$

D $\frac{78400}{33}$

Answer: D

Explanation:

We are given that, $T_n = 1 - \frac{3}{n} = \frac{n-3}{n}$

$T_4 = \frac{1}{4}, T_5 = \frac{2}{5}, T_6 = \frac{3}{6}, T_7 = \frac{4}{7}, ... T_{50} = \frac{47}{50}$

Hence, $\prod_{n=4}^{n=50} T_n = T_4 * T_5 * T_6 * T_7 * ... * T_{50}$

$\Rightarrow \prod_{n=4}^{n=50} T_n = \frac{1}{4} * \frac{2}{5} * \frac{3}{6} * \frac{4}{7} * ... * \frac{47}{50}$

$\Rightarrow \prod_{n=4}^{n=50} T_n = \frac{1 * 2 * 3}{48 * 49 * 50}$

Similarly, $\prod_{n=51}^{n=100} T_n = T_{51} * T_{52} * T_{53} * T_{54} * ... * T_{100}$

$\Rightarrow \prod_{n=51}^{n=100} T_n = \frac{48}{51} * \frac{49}{52} * \frac{50}{53} * \frac{51}{54} * ... * \frac{97}{100}$

$\Rightarrow \prod_{n=51}^{n=100} T_n = \frac{48 * 49 * 50}{98 * 99 * 100}$

Therefore, $\frac{\prod_{n=51}^{n=100} T_n}{\prod_{n=4}^{n=50} T_n} = \frac{\frac{48 * 49 * 50}{98 * 99 * 100}}{\frac{1 * 2 * 3}{48 * 49 * 50}} = \frac{48^2 * 49^2 * 50^2}{1 * 2 * 3 * 98 * 99 * 100} = \frac{78400}{33}$. Hence option D is the correct answer.

Question 47

If the population of a town is p in the beginning of any year then it becomes 3 + 2p in the beginning of the next year. If the population in the beginning of 2019 is 1000, then the population in the beginning of 2034 will be

A $(1003)^{15} + 6$

B $(997)^{15} - 3$

C $(997)2^{14} + 3$

D $(1003)2^{15} - 3$

Answer: D

Explanation:
The population of town at the beginning of 1st year = p

The population of town at the beginning of 2nd year = 3+2p

The population of town at the beginning of 3rd year = 2(3+2p)+3 = 2*2p+2*3+3 =4p+3(1+2)

The population of town at the beginning of 4th year = 2(2*2p+2*3+3)+3 = 8p+3(1+2+4)

Similarly population at the beginning of the nth year = 2^{n-1}p+3$(2^{n-1}-1)$ = $2^{n-1}(p+3)$-3

The population in the beginning of 2019 is 1000, then the population in the beginning of 2034 will be $(2^{2034-2019})(1000+3)$-3 = $2^{15}(1003)$-3

Question 48

An infinite geometric progression has the sum 10. Sum of all the possible integral values of the first term is

Answer:180

Explanation:
Let us consider the first term of GP=a

Common ratio=r

$\frac{a}{1-r}$=10

r=1-$\frac{a}{10}$

-1 < r< 1

-1 < 1-$\frac{a}{10}$ < 1

0 < a < 20 but the value of a cannot be 10

Sum of possible integral values of a =Sum of first 19 Natural Numbers - 10

= $\frac{19*20}{2}$ - 10

=180

Question 49

In a beaker 1 bacteria is kept at t=0 sec. Immediately before every second, the bacteria triples itself and immediately after every 2 seconds a disinfectant kills 1/3 of the bacteria. What is the number of bacteria in the beaker at t = 2019 s?

A 2^{1009} x 3^{1010}

B 2^{2019} x 3^{2019}

C 2^{1010} x 3^{1010}

D 2^{1009} x 3^{1009}

Answer: A

Explanation:
Number of bacteria in the beaker at **t = 0**, sec = 1 bacteria

Total number of bacteria in the beaker after 1 sec = 1 x 3= 3 bacteria

Total number of bacteria after 2 sec= Number of bacteria after multiplying - bacteria killed by disinfectant. = 3 x 3 - 3 x 3/3 = 3 x 2 = 6 bacteria

Number of bacteria killed by disinfectant at 2nd sec = **3.**

Total number of bacteria after 3 sec =6 x 3=18 bacteria

Total number of bacteria after 4 sec =18 x 2= 36 bacteria

......

So the number of bacteria becomes 6 times at the end of every two seconds.

Till t=2018, the number of bacteria in the beaker will be = 1x 6^{1009}=2^{1009} x 3^{1009}

At t=2019 the number of bacteria will triple, thus the number of bacteria will be = 2^{1009} x 3^{1010}

Hence A is correct.

Question 50

In an India-Australia match, Sachin hits a six such that the ball makes a semicircular trajectory. The ball then bounces multiple times, making similar trajectories, before coming to halt. If the total distance travelled by the ball is 990 meters and the length of each semicircular path, starting from the second, reduces by two-thirds of the previous bounce, then find out the shortest distance (in meters) between Sachin and the final position of the ball. Take $\pi = \frac{22}{7}$

A 315

B 630

C 345

D 600

Answer: B

Explanation:
Let, the radius of the first trajectory = r
Distance travelled by the ball in this trajectory = $\pi * r$
∵ length of each semicircular path reduces by $\frac{2}{3}rd$
∴ the above forms a GP with common ratio, a = $\frac{1}{3}$

Given, Sum of all paths = 990

=>$\frac{\pi * r}{1-a}$ = 990

=>$\frac{\pi * r}{1-\frac{1}{3}}$ = 990

=>r = 210

Shortest distance = Sum of all diameters of the semi-circular paths
= $\frac{2*210}{1-\frac{1}{3}}$ = 630 meters

Hence, option B is the correct answer.

Question 51

Sum of two non co-prime numbers a, b and their HCF gives 77. What is the number of possible values of (a, b)?

Answer:6

Explanation:
Let the HCF be h, then the numbers can be expressed as, a=hx, b=hy , where x,y are co-primes.

h+hx+hy = 77
h(1+x+y) = 77
h can be 1 or 7 or 11
h $\neq$ 1 (as a,b are non co-primes)
If h = 7, (1+x+y) = 11
x+y = 10
Now we have to select the values of x,y such that they are co-prime to each other.
x = 1, y = 9
x = 3, y = 7
x = 7, y = 3,

x = 9, y = 1
Hence when HCF is 7, there are 4 possible pairs of (x, y)
If HCF = 11, (1+x+y) = 7
x+y = 6
x = 1, y = 5
x = 5, y = 1
There are two possible values of (x,y).

Total values of a and b = 4+2=6
Hence 6 is the correct answer.

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Question 52

While multiplying three real numbers, Ashok took one of the numbers as 73 instead of 37. As a result, the product went up by 720. Then the minimum possible value of the sum of squares of the other two numbers is

Answer:40

Explanation:
We know that one of the 3 numbers is 37.
Let the product of the other 2 numbers be x.
It has been given that 73x-37x = 720
36x = 720
x = 20

Product of 2 real numbers is 20.
We have to find the minimum possible value of the sum of the squares of the 2 numbers.
Let x=a*b
It has been given that a*b=20

The least possible sum for a given product is obtained when the numbers are as close to each other as possible.
Therefore, when a=b, the value of a and b will be $\sqrt{20}$.

Sum of the squares of the 2 numbers = 20 + 20 = 40.

Therefore, 40 is the correct answer.

Question 53

Ram writes down all positive integers consecutively, starting from 1. He skips every multiple of 10 in the process. What is 3000th digit written by him?

A 3

B 7

C 9

D 1

Answer: D

Explanation:
Ram skips every 10th number hence he will skip 10,20,...

So one digit numbers written by Ram = 9 {From 1 to 9}

Two digit numbers written by Ram = 9*9 = 81 {From 11 to 99}

Three digit numbers written by Ram = 9*10*9 = 810 {From 101 to 999}

Exhausting all 3 digit number we will start with four digit number starting from 1001

Four digit numbers from 1001 to 1099 = 9*10 = 90

Four digit numbers from 1101 to 1109 = 9

Total number of digits so far = (1*9) + (2*81) + (3*810) + (4*99) = 2997

3rd digit of next number will be the 3000th digit.

Next number written = 1111

Hence we can say that digit 1 is 3000th digit that is written by Ram.Option D is the correct answer.

Question 54

How many pairs (m, n) of positive integers satisfy the equation $m^2 + 105 = n^2$?

Answer:4

Explanation:
$n^2 - m^2 = 105$

(n-m)(n+m) = 1*105, 3*35, 5*21, 7*15, 15*7, 21*5, 35*3, 105*1.

n-m=1, n+m=105 ==> n=53, m=52

n-m=3, n+m=35 ==> n=19, m=16

n-m=5, n+m=21 ==> n=13, m=8

n-m=7, n+m=15 ==> n=11, m=4

n-m=15, n+m=7 ==> n=11, m=-4

n-m=21, n+m=5 ==> n=13, m=-8

n-m=35, n+m=3 ==> n=19, m=-16

n-m=105, n+m=1 ==> n=53, m=-52

Since only positive integer values of m and n are required. There are 4 possible solutions.

Question 55

After distributing the sweets equally among 25 children, 8 sweets remain. Had the number of children been 28, 22 sweets would have been left after equally distributing. What is the smallest possible total number of sweets ?

A 328

B 348

C 358

D Data inadequate

Answer: C

Explanation:
Let the total number of chocolates be C

let chocolates received by each child = x

After distributing the sweets equally among 25 children, the number of chocolates left = 8
25x+8=C - Eq 1

Had there been 28 children, the number of chocolates that would be required = 22

25x+8-22 should be divisible by 28 - Eq 2

25x-14 should be divisible by 28

28x-(3x+14) should be divisible by 28

(3x+14) should be divisible by 28.

==> x=14

The total number of chocolates = 25x+8 = 25*14+8=358

C is the correct answer.

Question 56

How many numbers will divide 2460 and 2640 leaving the same remainder?

A 15

B 22

C 18

D 21

Answer: C

Explanation:
Suppose a number K leaves the same remainder R when it divides 2460 and 2640.
So, 2460 - R = K*p for some natural number p
Similarly, 2640 - R = K*q for some natural number q
When we subtract the two equations, we get 2640 - 2460 = K*(q-p) or 180 = K*(q-p)
Hence, all the numbers which are factors of the difference (180) between the given numbers will divide both the numbers leaving the same remainder.
So, all the factors of (2640 - 2460) = 180 will be such numbers.
So, we have to find the numbers of factors of 180.
$180 = 2^2 * 3^2 * 5$
Number of factors = (2 + 1)(2 + 1)(1 + 1) = 18
So, there are 18 such numbers which will divide 2460 and 2640 leaving the same remainder.
Hence, option c is the correct answer.

Question 57

N is a positive integer such that (N+1) is a factor of N!. If it is known that $1 \leq N \leq 100$, then how many values of 'N' are possible?

A 76

B 75

C 74

D 73

Answer: D

Explanation:
From Wilson's theorem, we know that
(p - 1)! mod p = p-1, where p is a prime number.
So when (N+1) will be a prime number than N! will not be divisible by (N+1).There are 26 prime numbers from 1 to 101. Hence for these 26 values, (N+1) will not be a factor of N!. Moreover, 3! is also not divisible by 4. Hence when N = 3, then also the given statement does not hold true. For all other values of N, N! will be divisible by (N+1). There are 100 numbers from 1 to 100. From these, for 27 numbers can be excluded. Thus, for 73 values of 'N', the given relation will hold true.

Question 58

What is the number of positive integral solutions to the equation
$[\frac{a}{49}] = [\frac{a}{51}]$ **where [x] represents the greatest integer less than or equal to x.**

A 623

B 624

C 625

D None of these

Answer: B

Explanation:
For a = 1,2,3,....,48 the value of $[\frac{a}{49}] = [\frac{a}{51}]$ = 0. The number of values is 48.
For a = 51,52,....,97 the value of $[\frac{a}{49}] = [\frac{a}{51}]$ = 1. The number of values is 47.
For a = 102,103,....,146 the value of $[\frac{a}{49}] = [\frac{a}{51}]$ = 2. The number of values is 45.
For a = 153,154,....,195 the value of $[\frac{a}{49}] = [\frac{a}{51}]$ = 3. The number of values is 43.
.
.
.
Hence the total number of solutions = 48+47+45+43+....+1

1 + 3 + 5 + . . . + 43 + 45 + 47 is an AP with a common difference of 2, first term 1 and number of terms = 24.

Sum = n/2 * (2a + (n-1)d) = 24/2 * (2 + 23*2) = 576

Hence, total number of solutions = 48 + 576 = 624

Question 59

The LCM of three natural numbers P, Q and R is $2^6 * 3^7 * 5^5 * 7^7$. If the LCM of 2P, Q and 6R is also $2^6 * 3^7 * 5^5 * 7^7$, then find out the total number of values that Q can take?

Answer:384

Explanation:
Given that, LCM of (P, Q, R) = $2^6 * 3^7 * 5^5 * 7^7$

LCM of (2P, Q, 6R) = $2^6 * 3^7 * 5^5 * 7^7$

We can see that we have multiplied by 2 in each of P and R even after that the power of 2 remains same in the LCM. This explains that the number of 2 in LCM depends upon the power of 2 available Q. Hence, we can say that Q is a multiple of 2^6.

We have multiplied by 3 in R even after that power of 3 remains same in the LCM. This explains that the number of 3 in LCM depends upon the the power of 3 available in either P or Q. So, number of 3 can be anything in Q ranging from 0 to 7.

There is no restriction on number of 5 and 7 repeating in Q.

Therefore, Q = $2^6 * 3^a * 5^b * 7^c$ {Where 0 $\leq$ a $\leq$ 7, 0 $\leq$ b $\leq$ 5, 0 $\leq$ c $\leq$ 7}

Hence, we can say that total number of values that Q can take = 1*8*6*8 = 384.

Question 60

N is a multiple of 72. Also, it is known that all the digits of N are different. What is the largest possible value of N?

Answer:9876543120

Explanation:
We know that the number is a multiple of 72. Therefore, the number must be a multiple of both 8 and 9.
If the number contains all the digits from 0 to 9, the sum of the digits will be 9*10/2 = 45.
Therefore, a number containing all the ten digits will be divisible by 9.
For a number to be divisible by 8, the last 3 digits should be divisible by 8.
Now, we have to find the largest possible number. Therefore, the left-most digit must be 9, the second digit from the left should be 8 and so on.
We'll get 9876543210 as the number. However, the last 3 digits are not divisible by 8. We must try to make the last 3 digits divisible by 8 without altering the position of other numbers. '120' is divisible by 8.

Therefore, the largest number with different digits divisible by 72 is 9876543120.

Question 61

The cost price of two scooters is in the ratio of 3:4. Ajay made Rs. 1 lakh and Rs. 2 lakh profit on each of them respectively. Which of the following can't be the ratio of their selling price?

A 41:77

B 53:81

C 37:49

D 61:97

Answer: C

Explanation:
The ratio of their cost price is 3:4 and the ratio of profit is 1:2.
So, the ratio of the selling price will be in between 1:2 and 3:4
Of the given options, only 37:49 is not in between the two ratios

Question 62

John borrowed Rs. 2,10,000 from a bank at an interest rate of 10% per annum, compounded annually. The loan was repaid in two equal instalments, the first after one year and the second after another year. The first instalment was interest of one year plus part of the principal amount, while the second was the rest of the principal amount plus due interest thereon. Then each instalment, in Rs., is

Answer:121000

Explanation:
We have to equate the installments and the amount due either at the time of borrowing or at the time when the entire loan is repaid. Let us bring all values to the time frame in which all the dues get settled, i.e, by the end of 2 years.

John borrowed Rs. 2,10,000 from the bank at 10% per annum. This loan will amount to 2,10,000*1.1*1.1 = Rs.2,54,100 by the end of 2 years.
Let the amount paid as installment every year be Rs.x.

John would pay the first installment by the end of the first year. Therefore, we have to calculate the interest on this amount from the end of the first year to the end of the second year. The loan will get settled the moment the second installment is paid.

=> 1.1x + x = 2,54,100
2.1x = 2,54,100
=> x = Rs. 1,21,000.

Therefore, 121000 is the correct answer.

Question 63

The owner of an art shop conducts his business in the following manner: every once in a while he raises his prices by X%, then a while later he reduces all the new prices by X%. After one such updown cycle, the price of a painting decreased by Rs. 441. After a second up-down cycle the painting was sold for Rs. 1,944.81. What was the original price of the painting?

A Rs. 2,756.25

B Rs. 2,256.25

C Rs. 2,500

D Rs. 2,000

Answer: A

Explanation:
Let the price of the painting be P
One cycle of price increase and decrease reduces the price by $x^2/100 * P = 441$
Let the new price be N => $P - x^2/100 * P = N$
Price after the second cycle = $N - x^2/100 * N$ = 1944.81
=> $(P - x^2/100 * P)(1 - x^2/100) = 1944.81$
=> $(P - 441)(1 - 441/P) = 1944.81$
=> $P - 441 - 441 + 441^2/P = 1944.81$
=> $P^2 - (882 + 1944.81)P + 441^2 = 0$
=> $P^2 - 2826.81P + 441^2 = 0$
From the options, the value 2756.25 satisfies the equation.
So, the price of the article is Rs 2756.25

Quantitative Aptitude for CAT Questions (download pdf)

Question 64

An ice-cream vendor can sell 100 ice-cream bricks for Rs.800 each. He realizes that he can sell 50 more bricks for every 25 rupees he reduces in the selling price of the ice-cream brick. What should be his selling price if he wants to maximize revenue? (The answer must be a multiple of 25)

Answer:425

Explanation:
The revenue can be written as a function of quantity into price.
Revenue= (800-25x)(100+50x)
Taking 25 common from both the brackets, we get 25*25*(32-x)(4+2x) = 625*(32-x)(4+2x)

Again, we can take 2 in common from the term in the second bracket, so revenue=1250(32-x)(2+x)

We need to maximize this. So, we need to maximise (32-x)(x+2)

=$32x + 64 - x^2 - 2x$

=$64 + 30x - x^2$

There are two methods in which we can find the maximum value of x.

Method 1: Differentiation

Differentiating $64 + 30x - x^2$ and equating with 0 we get 2x=30 so, x=15

Method 2: Completion of squares

$64 + 30x - x^2$ can be rewritten as $289 - [225 - 30x + x^2]$ = $289 - (x - 15)^2$

We must maximise $289 - (x - 15)^2$

$(x - 15)^2$ will always be positive, the minimum value it can take is when x=15, $(x - 15)^2$=0

So, $289 - (x - 15)^2$ is maximum when x=15

Hence, price=800-25*15

=800-375

Price=425

Question 65

A computer is sold either for Rs.19200 cash or for Rs.4800 cash down payment together with five equal monthly installments. If the rate of interest charged is 12% per annum, then the amount of each installment (nearest to a rupee) is:

A Rs.2880

B Rs.2965

C Rs.2896

D Rs.2990

E Rs.3016

Answer: B

Explanation:
Amount on which interest will be charged = 19200 - 4800 = 14400

The total amount paid will be equal to the sum of all monthly instalments. Therefore, we have

$14400 * k^{5a} = I(k^{4a} + k^{3a} + k^{2a} + k^a + 1)$ .....(1)

where, k = $1 + \frac{12}{100}$ & a = $\frac{1}{12}$

We know that, $k^{5a} - 1 = (k-1)(k^{4a} + k^{3a} + k^{2a} + k^a + 1)$

=>$k^{4a} + k^{3a} + k^{2a} + k^a + 1$ = $\frac{k^{5a}-1}{k-1}$ ....(2)

Substituting in equation (1) we get

I = $14400 * k^{5a} * \left[\frac{k-1}{k^{5a}-1}\right]$ ....(3)

On substituting the values of k and a in equation (3) we get
I $\approx$ 2965

Hence, option B.

Question 66

Gopal borrows Rs. X from Ankit at 8% annual interest. He then adds Rs. Y of his own money and lends Rs. X+Y to Ishan at 10% annual interest. At the end of the year, after returning Ankit's dues, the net interest retained by Gopal is the same as that accrued to Ankit. On the other hand, had Gopal lent Rs. X+2Y to Ishan at 10%, then the net interest retained by him would have increased by Rs. 150. If all interests are compounded annually, then find the value of X + Y.

Answer:4000

Explanation:

Amount of interest paid by Ishan to Gopal if the borrowed amount is Rs. (X+Y) = $\frac{10}{100} * (X+Y)$ = 0.1(X+Y)

Gopal also borrowed Rs. X from Ankit at 8% per annum. Therefore, he has to return Ankit Rs. 0.08X as the interest amount on borrowed sum.

Hence, the interest retained by gopal = 0.1(X+Y) - 0.08X = 0.02X + 0.1Y ... (1)

It is given that the net interest retained by Gopal is the same as that accrued to Ankit.

Therefore, 0.08X = 0.02X + 0.1Y

$\Rightarrow$ X = (5/3)Y ... (2)

Amount of interest paid by Ishan to Gopal if the borrowed amount is Rs. (X+2Y) = $\frac{10}{100} * (X+2Y)$ = 0.1X+0.2Y

In this case the amount of interest retained by Gopal = 0.1X+0.2Y - 0.08X = 0.02X + 0.2Y ... (3)

It is given that the interest retained by Gopal increased by Rs. 150 in the second case.

$\Rightarrow$ (0.02X + 0.2Y) - (0.02X + 0.1Y) = 150

$\Rightarrow$ Y = Rs. 1500

By substituting value of Y in equation (2), we can say that X = Rs. 2500

Therefore, (X+Y) = Rs. 4000.

Question 67

A metal trader sells zinc, copper and iron. On a particular day, the cost price of iron is 25% more than copper which in turn has cost price 33.33% more than zinc. The profit booked on zinc, copper and iron is 50%, 40% and 30% respectively. If the overall profit is 40%, what should be the ratio of quantities of zinc and iron sold on that day assuming that every metal was traded?

A 5/3

B 3/5

C 5/4

D Cannot be determined

Answer: A

Explanation:
Assuming the cost price of zinc/kg = a
Hence the price of copper/kg = 4a/3
Hence the price of iron/kg =5a/3

Assuming a/3 = b, The price per kg for zinc, iron and copper be 3b, 4b and 5b respectively.
The profit made on zinc/kg = 3b*0.5 = 1.5b
Profit on copper/kg = 4b*0.4 = 1.6b
Profit on iron/kg = 5b*0.3 =1.5b

Now assuming the quantities for zinc, iron and copper be x, y and z respectively.

Overall profit = (1.5bx+1.6by+1.5bz)/(3bx+4by+5bz) = 40/100 =0.4

=> (1.5bx+1.6by+1.5bz) = 1.2bx + 1.6by + 2bz

=> 0.3x = 0.5z

=> x/z =5/3

Question 68

A person earned an interest of 'S' when he invested a principal at simple interest for 3 years. He earned an interest of 'C' when he invested the same principal at compound interest for 2 years. The rate of interest was same in both the cases. If the ratio S:C = 15:11, then which of the following can be the rate of interest?

A 5%

B 10%

C 15%

D 20%

E 25%

Answer: D

Explanation:
Let the rate of interest be '100r %', Principal be 'P'.
S = P*3*r = 3Pr
C = $P(1+r)^2 - P = P(r^2+2r)$
S:C = 15:11
=> $\frac{3Pr}{P(r^2+2r)} = \frac{15}{11}$
=> $15r^2 + 30r = 33r$
=> r = 0.2
Rate of interest is 100 X .2 = 20%.

Question 69

Hari invested Rs 1000 in a bank, at 20% per annum, compounded annually, for three years. He withdrew Rs. 100 each at the end of the first and second years. He closed the investment at the end of three years and received Rs. T. If he had not taken Rs. 100 at the end of the first and second years, he would have received Rs. S at the end of three years. What is the value of S-T?

Answer:264

Explanation:
Hari took Rs 100 out at the end of the first year, with the interest compounded on this Rs 100 will value 1.20 x 1.20 x 100= Rs 144.
Hari took Rs 100 out at the end of the second year, With the interest compounded on this, Rs 100 will value 1.20 x 100= Rs 120.
Rupees he would have received if he had not taken out the money every year =144+120=264

Question 70

If a seller gives a discount of 15% on retail price, she still makes a profit of 2%. Which of the following ensures that she makes a profit of 20%?

A Give a discount of 5% on retail price.

B Give a discount of 2% on retail price.

C Increase the retail price by 2%.

D Sell at retail price.

Answer: D

Explanation:
Let the retail price be M and cost price be C.

Given,

0.85 M = 1.02 C

M = 1.2 C

If he wants 20% profit he has to sell at 1.2C, which is nothing but the retail price.

Question 71

For how many integral values of 'k' does the quadratic equation $y = |x^2 + kx + 7|$ **and the line** $y = 2$ **intersect at exactly 3 points ?**

A 4

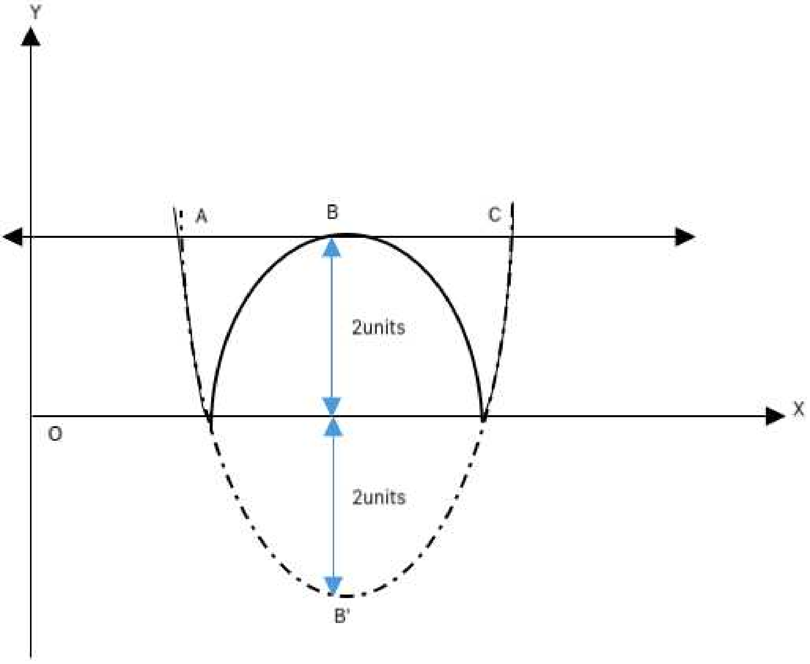
B 1

C 2

D 0

Answer: C

Explanation:



The graph of the equation $|x^2+kx+7|=0$ looks as shown above.

The equation $ax^2+bx+c=0$ can be written as $\left(x+\frac{b}{2a}\right)^2+\frac{(4ac-b^2)}{4a^2}$

The minimum value of the above equation is $\frac{-D}{4a^2}$ and occurs at x= $-\frac{b}{2a}$

$\therefore$ The coordinates for point B' is $\left(\frac{-b}{2a}, \frac{-D}{4a^2}\right)$ for equation of the form $ax^2+bx+c=0$.
$\because$ the line $y=2$ and equation $|x^2+kx+7|=0$ intersect at 3 points two of them being points A & C. The third point has to be the reflection of point B' such that the perpendicular distance from the x-axis will be 2 units.
$\therefore$ $|\frac{-D}{4a^2}|$= 2 where D$\geq$0

=> $|\frac{k^2-28}{4}|$ = 2

=>k^2 = 36 or 20
But, for k^2=20 => D<0
=> k = ± 6

$\therefore$ we have 2 such values.
Hence, option C is correct.

Question 72

The product of the distinct roots of $|x^2-x-6|=x+2$ is

A −16

B -4

C -24

D -8

Answer: A

Explanation:
We have, $|x^2-x-6|=x+2$

=> |(x-3)(x+2)|=x+2

For x<-2, (3-x)(-x-2)=x+2

=> x-3=1 =>x=4 (Rejected as x<-2)

For -2$\leq$ x<3, (3-x)(x+2)=x+2 =>x=2,-2

For x$\geq$ 3, (x-3)(x+2)=x+2 =>x=4

Hence the product =4*-2*2=-16

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Question 73

If a and b are integers such that $2x^2 - ax + 2 > 0$ and $x^2 - bx + 8 \geq 0$ for all real numbers x, then the largest possible value of $2a - 6b$ is

Answer:36

Explanation:
Let f(x) = $2x^2 - ax + 2$. We can see that f(x) is a quadratic function.

For, f(x) > 0, Discriminant (D) < 0

$\Rightarrow (-a)^2 - 4*2*2 < 0$

$\Rightarrow$ (a-4)(a+4)<0

$\Rightarrow$ a ϵ (-4, 4)

Therefore, integer values that 'a' can take = {-3, -2, -1, 0, 1, 2, 3}

Let g(x) = $x^2 - bx + 8$. We can see that g(x) is also a quadratic function.

For, g(x)≥0, Discriminant (D) $\leq$ 0

$\Rightarrow (-b)^2 - 4*8*1 < 0$

$\Rightarrow (b - \sqrt{32})(b + \sqrt{32}) < 0$

$\Rightarrow$ b ϵ (-$\sqrt{32}$, $\sqrt{32}$)

Therefore, integer values that 'b' can take = {-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5}

We have to find out the largest possible value of $2a - 6b$. The largest possible value will occur when 'a' is maximum and 'b' is minimum.

a_{max} = 3, b_{min} = -5

Therefore, the largest possible value of $2a - 6b$ = 2*3 - 6*(-5) = 36.

Question 74

f(x) is a four-degree polynomial and the coefficient of x^4 is 1.
If f(1) = 5, f(2) = 10, f(3) = 15 and f(4) = 20 then, what is the value of f(5)?

A 25

B 30

C 49

D 34

E 23

Answer: C

Explanation:
Let the polynomial be
f(x) = k(x - 1)(x - 2)(x - 3)(x - 4) + 5x
Since, it is given that the coefficient of x^4 is 1, k = 1
Putting x = 5, we get
f(5) = 4 * 3 * 2 * 1 + 25 = 29
Hence, option C is the correct answer.

Question 75

For what value of a, will the equation $(a^2-9)x^2-(a^2+2a-3)x+a^2+a-6=0$ have more than two solutions?

A -3

B 2

C 1

D No such value of a exists

Answer: A

Explanation:
The given equation will have more than two solutions when it is an identity.
=> $(a^2-9)=(a^2+2a-3)=(a^2+a-6)=0$
$(a^2-9)=0$ => $a=3$ or -3
$(a^2+2a-3)=0$ => $a=-3$ or 1
$(a^2+a-6)=0$ => $a=-3$ or 2
Since, for $a=-3$ satisfies all the equations, for $a=-3$, the given equation will have more than two solutions or infinite solutions.
Hence, option A is the correct answer.

Question 76

An equation $5005x^4-bx^3+cx^2-dx+3774=0$ has roots α,β,γ and δ and such that each of the roots is in the form of $\frac{p}{q}$ where p and q are prime numbers. How many unordered quadruplets of roots α,β,γ, and δ are possible ?

A 32

B 24

C 16

D 12

Answer: B

Explanation:
We have 5005 = 5 x 7 x 11 x 13 and 3774 = 2 x 3 x 17 x 37
The product of roots for the given polynomial is given by = $\frac{3774}{5005}=\frac{2\times3\times17\times37}{5\times7\times11\times13}$

Since, all the roots are of the for $\frac{p}{q}$ where $p,q\neq1$ and both are prime.
$\therefore$ the roots should be such that the numerator must be one among 2, 3, 17 or 37 and the denominator must be one among 5, 7, 11 or 13.
Also, as each of the numbers is prime, so they will always be co-prime to each other.

For '2' as numerator we can select one of the denominators in 4 ways.
For '3' as numerator we can select one out of the remaining three denominators in 3 ways.
Similarly, For '17' as numerator we can select one out of the remaining two denominators in 2 ways.
And lastly for '37' as numerator we can select one out of the remaining one denominator in 1 way.
As all the selections are mutually dependent, so we have the total number of possible quadruplets as = $4\times3\times2\times1=24$

Hence, option B is the correct choice.

Question 77

Which of the following is not a value that 'k' can take if it is known that the roots of the equation $kx^2-kx+12=0$ are real and rational?

A -16

B -27

C -64

D -50

Answer: C

Explanation:

Roots of a quadratic equation of the form $ax^2 - bx + c = 0$ are given by $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

$kx^2 - kx + 12 = 0$:

Substituting the values of a, b and c, we get,

Roots = $\frac{k\pm\sqrt{k^2-4k*12}}{2k}$
= $\frac{k\pm\sqrt{k^2-48k}}{2k}$

As we can see, for the roots to be real and rational, the value $\sqrt{k^2-48k}$ must be a perfect square.

Let us check the options:

-16:
$\sqrt{16^2-48*-16}$ = $\sqrt{16^2(1+3)}$ = $\sqrt{16*4}$ = 8

-27
$\sqrt{27^2-48*-27}$ = $\sqrt{27^2+48*27}$ = $\sqrt{27^2+16*3^4}$ = $\sqrt{3^4(3^2+16)}$
=$\sqrt{3^4*25}$

As we can see, -27 also yields a perfect square.

-50:
$\sqrt{50^2-48*-50}$ = $\sqrt{50(50+48)}$ = $\sqrt{50*98}$ = $\sqrt{100*49}$.
-50 also yields a perfect square.

-64
$\sqrt{64^2-48*-64}$ = $\sqrt{64^2+48*64}$ = $\sqrt{64(64+48)}$ = $64*112$
$112 = 16*7$
As we can see, 112 contains only one 7 and hence, -64 will not yield a perfect square value when substituted in k^2-48k. Therefore, the root will be irrational in this case and hence, k cannot be -64. Therefore, option C is the right answer.

Question 78

If $U^2+(U-2V-1)^2$= -$4V(U+V)$, then what is the value of $U+3V$?

A 0

B $\frac{1}{2}$

C $\frac{-1}{4}$

D $\frac{1}{4}$

Answer: C

Explanation:
Given that $U^2+(U-2V-1)^2$= -$4V(U+V)$

$\Rightarrow U^2 + (U - 2V - 1)(U - 2V - 1)$ = $-4V(U + V)$

$\Rightarrow U^2 + (U^2 - 2UV - U - 2UV + 4V^2 + 2V - U + 2V + 1)$ = $-4V(U + V)$

$\Rightarrow U^2 + (U^2 - 4UV - 2U + 4V^2 + 4V + 1)$ = $-4V(U + V)$

$\Rightarrow 2U^2 - 4UV - 2U + 4V^2 + 4V + 1 = -4UV - 4V^2$

$\Rightarrow 2U^2 - 2U + 8V^2 + 4V + 1 = 0$

$\Rightarrow 2[U^2 - U + \frac{1}{4}] + 8[V^2 + \frac{V}{2} + \frac{1}{16}] = 0$

$\Rightarrow 2(U - \frac{1}{2})^2 + 8(V + \frac{1}{4})^2 = 0$

Sum of two square terms is zero i.e. individual square term is equal to zero.

$U - \frac{1}{2}$ = 0 and $V + \frac{1}{4}$ = 0

U = $\frac{1}{2}$ and V = $-\frac{1}{4}$

Therefore, $U + 3V$ = $\frac{1}{2}$+ $\frac{-1*3}{4}$ = $\frac{-1}{4}$. Hence, option C is the correct answer.

Question 79

If a and b are roots of the equation $8x^2 - 7x + 1$, find the value of $4[a + b + a^2 + b^2 + a^3 + b^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\infty]$.

Answer:10

Explanation:
The given equation is $8x^2 - 7x + 1$

$a + b = \frac{7}{8}$

$ab = \frac{1}{8}$

We see that both a and b are greater than 0 and less than 1.

$4[a + b + a^2 + b^2 + a^3 + b^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\infty]$

Can be written as $4[(a + a^2 + a^3 .... + \infty) + (b + b^2 + b^3 ...... + \infty)]$

$4[\frac{a}{1-a} + \frac{b}{1-b}] = 4 * \frac{(a+b)-2ab}{1-(a+b)+ab}$

Substituting the values,

$4 * \frac{(a+b)-2ab}{1-(a+b)+ab}$ = 10

Question 80

If $\log_{12} 81 = p$, then $3\left(\frac{4-p}{4+p}\right)$ is equal to

A $\log_4 16$

B $\log_6 16$

C $\log_2 8$

D $\log_6 8$

Answer: D

Explanation:
Given that: $\log_{12} 81 = p$

$\Rightarrow \log_{81} 12 = \frac{1}{p}$

$\Rightarrow 4\log_3 3 * 4 = \frac{1}{p}$

$\Rightarrow 1 + \log_3 4 = \frac{4}{p}$

Using Componendo and Dividendo,

$\Rightarrow \frac{1 + \log_3 4 - 1}{1 + \log_3 4 + 1} = \frac{4-p}{4+p}$

$\Rightarrow \frac{\log_3 4}{2 + \log_3 4} = \frac{4-p}{4+p}$

$\Rightarrow \frac{\log_3 4}{\log_3 9 + \log_3 4} = \frac{4-p}{4+p}$

$\Rightarrow \frac{\log_3 4}{\log_3 36} = \frac{4-p}{4+p}$

$\Rightarrow 3 * \frac{4-p}{4+p} = \frac{3\log_3 4}{\log_3 36}$

$\Rightarrow 3 * \frac{4-p}{4+p} = \frac{\log_3 64}{\log_3 36}$

$\Rightarrow 3 * \frac{4-p}{4+p} = \log_{36} 64$

$\Rightarrow 3 * \frac{4-p}{4+p} = \log_{6^2} 8^2 = \log_6 8$. Hence, option D is the correct answer.

Question 81

If x is a real number such that $\log_3 5 = \log_5(2+x)$, then which of the following is true?

A 0 < x < 3

B 23 < x < 30

C x > 30

D 3 < x < 23

Answer: D

Explanation:
$1 < \log_3 5 < 2$
=> $1 < \log_5(2+x) < 2$
=> $5 < 2 + x < 25$
=> $3 < x < 23$

Question 82

It is given that 3log(x-2y)=log($2x^2$-5xy+$2y^2$)+log y, where x and y are positive real numbers. The value of log(x+2y)-logy is

A log7

B log3

C log5

D log2

Answer: A

Explanation:

3log(x-2y)=log($2x^2y$-$5xy^2$+$2y^3$)

$(x-2y)^3$ =($2x^2y$-$5xy^2$+$2y^2$)

x^3-$8y^3$-$6x^2y$+$12xy^2$=($2x^2y$-$5xy^2$+$2y^3$)

x^3-$10y^3$-$8x^2y$+$17xy^2$=0

Assuming x/y = k

We have, k^3-$8k^2$+17k-10=0

k=1,2 or 5

Hence, x=y,x=2y,x=5y (Since x/y =k)

In log y, y should be positive and hence x will be positive.

for x=y, x-2y will be negative

for x=2y, x-2y will be zero.

for x=5y, x-2y will be positive.

Also for x=5y, $2x^2$-5xy+$2y^2$ is positive

Hence only x=5y will satisfy.

log(x+2y)-logy = log(x/y +2) = log 7

(A) is the answer

Question 83

If $(5.55)^x = (0.555)^y = 1000$, then the value of $\frac{1}{x} - \frac{1}{y}$ is

A $\frac{1}{3}$

B 3

C 1

D $\frac{2}{3}$

Answer: A

Explanation:

We have, $(5.55)^x = (0.555)^y = 1000$

Taking log in base 10 on both sides,

x($\log_{10} 555$-2) = y($\log_{10} 555$-3) = 3

Then, x($\log_{10} 555$-2) = 3.....(1)

y($\log_{10} 555$-3) = 3 .....(2)

From (1) and (2)

=> $\log_{10} 555 = \frac{3}{x} + 2 = \frac{3}{y} + 3$

=> $\frac{1}{x} - \frac{1}{y} = \frac{1}{3}$

Question 84

What is the value of $0.05^{\log_{\sqrt{20}}(0.3+0.03+0.003+........\infty)}$?

A 6

B 9

C 3

D 1

Answer: B

Explanation:

$0.05^{\log_{\sqrt{20}}(0.3+0.03+0.003+........\infty)}$

= $\left(\frac{1}{20}\right)^{\log_{\sqrt{20}}(0.3+0.03+0.003+........\infty)}$

= $(20^{-1})^{2\log_{20}(0.3+0.03+0.003+........\infty)}$

= $20^{\log_{20}(0.3+0.03+0.003+........\infty)^{-2}}$

= $(0.3+0.03+0.003+........\infty)^{-2}$

= $3^{-2}(0.1+0.01+0.001+........\infty)^{-2}$

= $3^{-2}\left(\frac{1}{10}+\frac{1}{100}+.....\infty\right)^{-2}$

= $3^{-2} * \left(\frac{\frac{1}{10}}{1-\frac{1}{10}}\right)^{-2}$

= $3^{-2} * \left(\frac{1}{9}\right)^{-2}$

= 9

Hence, option B is the correct answer.

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Question 85

How many values of 'p' satisfy the following equation:
$(\log_5 p)^2 + \log_{5p}(5/p) = 1$**?**

A 1

B 2

C 3

D more than 3

Answer: C

Explanation:
$\log_{5p}(5/p) = \log_{5p} 5 - \log_{5p} p$
= $\frac{\log_5 5}{\log_5 5p} - \frac{\log_5 p}{\log_5 5p}$

$\log_5 5 = 1$ and $\log_5 5p = \log_5 5 + \log_5 p$ = 1 + $\log_5 p$

=> $\log_{5p}(5/p)$ = $\frac{1}{1+\log_5 p} - \frac{\log_5 p}{1+\log_5 p}$ = $\frac{1-\log_5 p}{1+\log_5 p}$

The equation can be written as:

$(\log_5 p)^2 + \frac{1-\log_5 p}{1+\log_5 p} = 1$
Let $\log_5 p$ = t
The equation becomes $t^2 + \frac{1-t}{1+t} = 1$
=> $t^3 + t^2 + 1 - t = 1 + t$
=> $t^3 + t^2 - 2t = 0$
Solving this, we get, t = 0, 1, -2
=> x = 1, 5, 1/25

Question 86

If $log_{70}7 = a$ and $log_{70}5 = b$, then the value of $log_{70}392$ is:

A 3 (1 + b - a)

B 3 - 3b - a

C 3 (1 - ab)

D 3 - 3a - b

Answer: B

Explanation:
$log_{70}392 = log_{70}49 + log_{70}8$

= $2log_{70}7 + log_{70}8$

= 2a + $log_{70}8$

$log_{70}8 = 3log_{70}2$
$3log_{70}2 = 3log_{70}(\frac{70}{35})$
$3log_{70}(\frac{70}{35}) = 3(1 - a - b)$
$log_{70}392 = log_{70}49 + log_{70}8$
= 2a + 3 - 3a - 3b = 3 - 3b - a

Question 87

If $\log_{24} 6 = x$ $\log_{12} 54 = y$ then

A $y = \frac{(4+x)}{(x-3)}$

B $y = \frac{(3-x)}{(x+2)}$

C $x = \frac{(3-y)}{(y-2)}$

D $x = \frac{(y+2)}{(8-y)}$

Answer: D

Explanation:
Changing into base 2, we get

x= (1+log3)/(3+log3)...(1)

y=$$/frac{1+3log3}{2+log3$$

=> log3 = $\frac{1-2y}{y-3}$ ......(2)

After putting the value of log3 in (1),

we get, x= $\frac{1+\frac{1-2y}{y-3}}{3+\frac{1-2y}{y-3}}$

x = $\frac{y-3+1-2y}{3y-9+1-2y}$

$x = \frac{(y+2)}{(8-y)}$

Hence, D is the answer.

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Question 88

Two functions A(x) and B(x) are such that $4A^2(x) - 2B(x)B(-x) = B^2(x) + B^2(-x)$. If $A(4) = 24$ what is the value of $A(-4)$?

A 24

B -24

C 32

D Cannot be determined

Answer: D

Explanation:

$4A^2(x) - 2B(x)B(-x) = B^2(x) + B^2(-x)$
$=> 4A^2(x) = [B(x) + B(-x)]^2$
$=> 4A^2(x) = 4A^2(-x)$
$=> A(x) = \pm A(-x)$
$=> A(-4) = \pm A(4) = \pm 24$

Question 89

If the domain of the function f(x) is [-100,700]. Two functions g and h are defined such that g(y) = f($2y^2 + 5$) and h(z) = g(z^2). Find the number of integral values for which h(z) is defined.

A 6

B 9

C 4

D 2

Answer: B

Explanation:

h(z) = g(z^2)
g(y) = f($2y^2 + 5$)
g(z^2) = f($2z^4 + 5$)
-100<=$2z^4 + 5$<=700
-52.5<=z^4<=347.5
z can take 9 values -4, -3, -2, -1, 0, 1, 2, 3, 4.

Question 90

If a function F(x) is defined such that F(x)+F($x-1$)=x^2 and F(10)=2019. Then the value of F(52).

Answer:3342

Explanation:
F(x)+F($x-1$)=x^2

F(11)+F(10)=11^2 ----- Eq (1)

F(12)+F(11)=12^2 ----- Eq (2)

Eq(2) - Eq(1)

F(12)-F(10)=12^2 - 11^2 ----- Eq (3)

F(13)+F(12)=13^2 ----- Eq (4)

F(14)+F(13)=14^2 ----- Eq (5)

Eq(5) - Eq(4)

F(14)-F(12)=14^2 - 13^2 ----- Eq (6)

Adding Eq (6) and Eq(3), we get

F(14) - F(10)=14^2 - 13^2 + 12^2 - 11^2

Similarly, F(52) - F(10) = 52^2 - 51^2 + 50^2 - 49^2.........+ 12^2 - 11^2

F(52) = $(52+51)(52-51)+(50+49)(50-49)+$ ....$(12+11)(12-11)$ + F(10)

=$52+51+50+49+$....................$12+11$+2019

=1323+2019

=3342
3342 is the correct answer.

How to prepare for Data Interpretation for CAT

Question 91

A quadratic function f(x) has a value 5 at x=-2. If the maximum value of the function is 8 at x=-1, then f(-3) is

A 3

B -6

C -4

D 5

Answer: C

Explanation:
Assuming the quadratic function to be f(x) = ax^2 +bx+c

We have, f(-2) = 4a-2b+c = 5

f(-1) = a-b+c = 8

=> 3a-b=-3 ....(1)

Now the maximum value will occur at -b/2a = -1

=> b=2a ....(2)

From (1) and (2), we get

3a-2a=-3 =>a=-3

b=3a+3 = -9+3=-6

c = 8+b-a = 8-6+3 = 5

Therefore f(-3) = 9a-3b+c = 9*-3-3*-6+5 = -4

Alternate Solution:

As the quadratic equation reaches its maximum when x=-1, it is of the form a(x+1)^2+c.
As this maximum value equals 8, the value of c = 8. Hence, the quadratic equation is of the form a(x+1)^2+8
The value of this function when x=-2 is 5. Hence, a+8=5 or a=-3
So, the quadratic equation is -3(x+1)^2+8.
When, x=-3, it equals -3*4+8 = -12+8 = -4

Question 92

Let f be a function such that f (mn) = f (m) f (n) for every positive integers m and n. If f (1), f (2) and f (3) are positive integers, f (1) < f (2), and f (24) = 54, then f (18) equals

Answer:12

Explanation:
Given, f(mn) = f(m)f(n)
when m= n= 1, f(1) = f(1)*f(1) ==> f(1) = 1

when m=1, n= 2, f(2) = f(1)*f(2) ==> f(1) = 1

when m=n= 2, f(4) = f(2)*f(2) ==> f(4) = $[f(2)]^2$

Similarly f(8) = f(4)*f(2) =$[f(2)]^3$

f(24) = 54

$[f(2)]^3$ * $[f(3)]$ = $3^3 * 2$

On comparing LHS and RHS, we get

f(2) = 3 and f(3) = 2

Now we have to find the value of f(18)

f(18) = $[f(2)]$ * $[f(3)]^2$

= 3*4=12

Question 93

If the domain of $\frac{x^3+3x^2-10x-24}{x^3+4x^2-11x-30} > 2$ is (x,y) what is |x+y|?

Answer:11

Explanation:
After factorising , the equation becomes
$\frac{(x+4)(x-3)(x+2)}{(x+2)(x-3)(x+5)}$ >2

$\frac{x+4}{x+5} > 2$

$\frac{x+4}{x+5} - 2 > 0$

$\frac{x+4}{x+5} - \frac{2(x+5)}{x+5}$ >0

$\frac{-(x+6)}{x+5}$ >0

$\frac{(x+6)}{x+5}$ <0

x>-6 and x < -5. Therefore the domain of x is (-5,-6)
|x+y|=11.

How to prepare for Quantitative aptitude for CAT

Question 94

If $|x^2-10|+|2x-17|+|x+4|\leq 36$ and a is the sum of the integral values that x can take and b is the number of integral values that x can take, find a^2+b^2

A 97

B 106

C 125

D 146

Answer: C

Explanation:
Consider $|x^2-10|$ since both the other parts of the equation are in modulus, they would be added to $|x^2-10|$. So, the maximum $|x^2-10|$ can be is when the other two parts are 0.
So, let $|x^2-10|\leq 36$
Then, the integral values that x can take are -6,-5,-4,-3,-2,-1, 0, 1, 2, 3, 4, 5, 6

Now, consider |2x-17| this would be minimum when x is closest to 2x-17=0
Or x=17/2. As can be seen from above, the maximum value that x can take is 6. As the value of x reduces from 6, |2x-17| will increase.

Now consider |x+4| this would be minimum when x=-4.

Let us consider the equation for $x\leq -4$ Then x can be=-4, -5, -6. The equation will be $x^2-10+17-2x+-4+-x\leq 36$
Only -4 satisfies the equation. (1 value)

For 4>x>-4 the equation will become $10-x^2+17-2x+4+x\leq 36$
Here 3, 2, 1, 0, -1, -2, -3 satisfy the equation. (7 such values)

For, x=4, 5, 6 the equation will become $x^2-10+17-2x+4+x\leq 36$
Here, 4 and 5 satisfy the equation. (2 values)

The sum of integral values that x can take is 5 (since the remaining terms cancel themselves out)
The number of values it can take is 1+7+2=10 values.
$5^2+10^2=125.$
So, Option C is the correct answer.

Question 95

Find the number of non-negative integral points that satisfy 2x + y > 16 and x + 2y = 20.

A 8

B 9

C 10

D 11

Answer: A

Explanation:
Number of non-negative integral points on x + 2y = 20 => (20,0), (18,1),......,(2,9) and (0,10) => 11 points
Intersection point of 2x + y = 16 and x + 2y = 20 is (4,8)
So, all the points among the above 11 points that have x coefficient less than or equal to 4 are removed.
=> 3 points are removed.
Hence, required number of points = 11 - 3 = 8

Question 96

Find the smallest integer value of 'n' such that for all m >= n, the value $m^3-16m^2+81m-126$ is positive.

Answer:8

Explanation:
The expression $m^3 - 16m^2 + 81m - 126$ can be written as $(m-3)(m-6)(m-7)$
(m - 3)(m - 6)(m - 7) >= 0 for m >= 7 U [3, 6].
So, the smallest integer value of 'n' such that for all m >= n, the value of (m - 3)(m - 6)(m - 7) is positive is n = 8.

How to prepare for Verbal Ability for CAT

Question 97

A row contains 16 seats. The number of ways in which 5 seats can be chosen such that no 2 seats are adjacent to each other is

Answer:792

Explanation:
We know that no 2 seats are adjacent to each other.
Let us place the 5 seats and then distribute the empty seats between them.
Once we choose the 5 seats, we have to ensure that no 2 seats are adjacent to each other.
There will be 4 gaps between the 5 seats.
Let us distribute 4 empty seats between them.
So, out of the 16 seats, we have accounted for 9 seats.
The remaining 16-9 = 7 empty seats must be distributed now.
There are 6 spaces available in total. We have to distribute these 7 seats in these 6 spaces.
Let 'a' be the number of empty seats to the left of the first seat, 'b' be the number of empty seats (excluding the one we already distributed) between the first and second chair and so on.

We know that a+b+c+d+e+f = 7.
The number of non-negative integral solutions of this equation is given by $(7+6-1)C_{6-1}$ = $12C_5$ = 792.
Therefore, 792 is the right answer.

Question 98

Four integers w,x,y,z (not necessarily unique) are selected at random from 1 to 1000 numbers both inclusive. . The probability that wz-xy is even is

A $\frac{1}{16}$

B $\frac{5}{8}$

C $\frac{9}{16}$

D $\frac{5}{16}$

Answer: B

Explanation:
wz-xy will be even only when both wz and xy are even or odd
The probability of wz being odd is $\frac{1}{2}*\frac{1}{2}$
Probability for wz -xy being odd is $(\frac{1}{4})^2$
Similarly, the probability of wz to be even is 1-probability of being odd= $\frac{3}{4}$
Probability for wz -xy being even is $(\frac{3}{4})^2$
Total probability=$\frac{1}{16}+\frac{9}{16}$
=$\frac{5}{8}$
B is the correct answer.

Question 99

In how many ways can Nihal select 3 apples from a row of 33 apples such that there should be at least 3 apples between any selected apples. Assume that all the apples are distinct.

A 4060

B 2925

C 6090

D 7140

Answer: B

Explanation:
Let p,X,q,Y,r,Z,s be the row of apples, where X, Y, Z be the apples which are selected.

Let's analyse the values of p, q, r, s.

p $\geq$ 0, q $\geq$ 3,r $\geq$ 3, s $\geq$ 0

p+q+r+s=30 (3 apples are already selected, hence 33-3=30)

p+q+3+r+3+s=30

p+q+r+s=24

Number of ways in which 24 distinct objects arranged in a row can be partitioned into 4 parts is $^{24+4-1}C_{4-1}$

=2925

B is the correct answer.

How to prepare for Logical Reasoning for CAT

Question 100

Payal has 7 P's and 8 Q's. In how many ways can she arrange them in a row such that it reads the same both forwards and backwards?

Answer:35

Explanation:
There are 7 P's and 8 Q's, the row will have 15 letters.

Payal wants to arrange them in a row such that it reads the same both forwards and backwards.
So it is enough if we arrange the first 8 letters.

Since there are odd number of P's , P will come in the middle.

_, _, _, _, _, _, _, P, _, _, _, _, _, _, _

The first 7 letters will be filled with 3P's and 4 Q's and the last 7 letters will be identical to the first 7 letters.

Number of ways in which 3P's and 4 Q's can be arranged is $\frac{7!}{3!*4!}$

=35

35 is the correct answer.

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